Assignment #2

Posted on February 26 – due by March 7, 5:30pm

Question 1: Hilarious Numbers

Let us define that a positive integer greater than 1 is called *hilarious* if the sum of its unique prime factors is a prime itself. Therefore, all primes are hilarious. Also, for example, 12 is hilarious, because its unique prime factors are 2 and 3, and their sum 5 is a prime, too. On the other hand, 84 is not hilarious, because $84 = 2^2 \cdot 3 \cdot 7$ and $2 + 3 + 7 = 12$, which is not a prime.

a) Write a program in Java, C, C++, Python, or pseudocode that finds for a given number $n$ the first $n$ hilarious numbers. Attach a printout of your program to your assignment.

b) What are the first 20 hilarious numbers as discovered by your program? If you used pseudocode, you need to find these numbers “by hand.”

Question 2: Prime Factor Examples

Write down the prime factorization (in ascending order) of each of the following integers (Example: $720 = 2^4 \cdot 3^2 \cdot 5$).

a) 258
b) 100000
c) 6250
d) 104

Question 3: Euclidean Algorithm

Use the Euclidean algorithm to determine the following greatest common divisors. Write down every step in your calculation.

a) $\text{gcd}(3300, 550)$
b) $\text{gcd}(177, 300)$
c) $\text{gcd}(912, 625)$
**Question 4: Matrices**

a) Find a matrix $M$ such that

$$
\begin{bmatrix}
2 & 4 \\
1 & 3
\end{bmatrix}
+ 
\begin{bmatrix}
-6 & 3 \\
0 & -2
\end{bmatrix}
= 
\begin{bmatrix}
4 & 27 \\
6 & 15
\end{bmatrix}
$$

Hint: For each column of $A$ you have to solve a system of linear equations. Write down all of these equations and every step of their solution.

b) Show that for any zero-one matrix $A$ it is true that

- $A \land A = A$
- $A \lor A = A$

**Question 5: Rules of Inference**

Use rules of inference to show whether the following arguments are valid or not:

a) If a course is boring and taught at a late time, the students will fall asleep. The course is boring, and the students do not fall asleep. Therefore, the course is not taught at a late time.

b) All UMB students are Red Sox fans. Whenever the Red Sox win, all Red Sox fans celebrate. The Red sox won. Therefore, Kermit, who is a UMB student, celebrates.

c) If time travel were possible, someone from the future would have visited us. If someone from the future had visited us, we would have heard about it. We have not heard about it. Therefore, time travel is impossible.

**Question 6: Proofs**

Prove or disprove the following statements:

a) $2^n + 1$ is prime for all positive integers $n$.

b) Prove that for any integer $x$, $x^2 + x + 1$ is odd.

c) For every integer $n$, $n(n + 1)$ is even.

d) The product of three odd integers is odd.
Question 7: Formalization of Logical Expressions

a) Franz and Erika are the only professors who play soccer.

b) All CS320 students know each other.

c) Petra and Bert never take classes that are interesting.

d) Any UMB student will fail if he/she does not complete Assignment #1.

e) Andreas beats up all professors that fail him and do not fail his sister Susanne.

f) There is at most one computer scientist who can dance.

g) There is one UMB student who is taller than 7 feet and taller than all other UMB students.

h) All UMB students who work full-time prefer evening classes.

i) There are no scientists who are neither crazy nor dangerous.

j) George and Claudia always attend the same courses.