Back to “Serious” Topics…

Knowledge Representation and Reasoning

Knowledge Representation & Reasoning

Knowledge representation is the study of how knowledge about the world can be represented and what kinds of reasoning can be done with that knowledge.

We will discuss two different systems that are commonly used to represent knowledge in machines and perform algorithmic reasoning:

- Propositional calculus
- Predicate calculus

Propositional Calculus

In propositional calculus,
- features of the world are represented by propositions,
- relationships between features (constraints) are represented by connectives.

Example:
LECTURE_BORING \land TIME_LATE \implies SLEEP
This expression in propositional calculus represents the fact that for some agent in our world, if the features LECTURE_BORING and TIME_LATE are both true, the feature SLEEP is also true.

Propositional Calculus

You see that the language of propositional calculus can be used to represent aspects of the world.

When there are
- a language, as defined by a syntax,
- inference rules for manipulating sentences in that language, and
- semantics for associating elements of the language with elements of the world,
then we have a system called logic.

The Language

Atoms:
The atoms T and F and all strings that begin with a capital letter, for instance, P, Q, LECTURE_BORING, and so on.

Connectives:
- \lor “or”
- \land “and”
- \implies “implies” or “if-then”
- \lnot “not”

Syntax of well-formed formulas (wffs):
- Any atom is a wff.
- If \( \omega_1 \) and \( \omega_2 \) are wffs, so are
  - \( \omega_1 \land \omega_2 \) (conjunction)
  - \( \omega_1 \lor \omega_2 \) (disjunction)
  - \( \omega_1 \implies \omega_2 \) (implication)
  - \( \lnot \omega_1 \) (negation)
- There are no other wffs.
The Language

- Atoms and negated atoms are called literals.
- In $\omega_1 \supset \omega_2$, $\omega_1$ is called the antecedent, and $\omega_2$ is called the consequent of the implication.
- Examples of wffs (sentences):
  
  - $(P \land Q) \supset \neg P$
  - $P \supset \neg P$
  - $P \lor P \supset P$
  - $(P \supset Q) \supset (\neg Q \supset \neg P)$
  - $\neg \neg P$

  - The precedence order of the above operators is $\neg \land \lor \supset$.
  For example, $\neg P \lor Q \supset R$ means $((\neg P) \lor Q) \supset R$.

Rules of Inference

We use rules of inference to generate new wffs from existing ones.

One important rule is called modus ponens or the law of detachment. It is based on the tautology $(P \land (P \supset Q)) \supset Q$. We write it in the following way:

$P$

$P \supset Q$

\[ \therefore Q \]

The two hypotheses $P$ and $P \supset Q$ are written in a column, and the conclusion below a bar, where $\therefore$ means "therefore".

Proofs

The sequence of wffs $\{\omega_1, \omega_2, \ldots, \omega_n\}$ is called a proof (or a deduction) of $\omega_n$ from a set of wffs $\Delta$ if (and only if) each $\omega_i$ in the sequence is either in $\Delta$ or can be inferred from one or more wffs earlier in the sequence by using one of the rules of inference.

If there is a proof of $\omega_n$ from $\Delta$, we say that $\omega_n$ is a theorem of the set $\Delta$. We use the following notation:

$\Delta \vdash \omega_n$

In this notation, we can also indicate the set of inference rules $R$ that we use:

$\Delta \vdash R \omega_n$

Semantics

- In propositional logic, we associate atoms with propositions about the world.
- We thereby specify the semantics of our logic, giving it a "meaning".
- Such an association of atoms with propositions is called an interpretation.
- In a given interpretation, the proposition associated with an atom is called the denotation of that atom.
- Under a given interpretation, atoms have values – True or False. We are willing to accept this idealization (otherwise: fuzzy logic).
Introduction to Artificial Intelligence
Lecture 12: Knowledge Representation & Reasoning I

Semantics

Example:
"Gary is either intelligent or a good actor.
If Gary is intelligent, then he can count from 1 to 10.
Gary can only count from 1 to 2.
Therefore, Gary is a good actor."

Propositions:
I: "Gary is intelligent."
A: "Gary is a good actor."
C: "Gary can count from 1 to 10."

Semantics

Step 1: ¬C Hypothesis
Step 2: I ⊃ C Hypothesis
Step 3: ¬I Modus Tollens Steps 1 & 2
Step 4: A ∨ I Hypothesis
Step 5: A Disjunctive Syllogism Steps 3 & 4

Conclusion: A ("Gary is a good actor.")

Semantics

- In propositional logic, we associate atoms with propositions about the world.
- We thereby specify the semantics of our logic, giving it a "meaning".
- Such an association of atoms with propositions is called an interpretation.
- In a given interpretation, the proposition associated with an atom is called the denotation of that atom.
- Under a given interpretation, atoms have values – True or False. We are willing to accept this idealization (otherwise: fuzzy logic).

Semantics

- If no interpretation satisfies a wff (or a set of wffs), the wff is said to be inconsistent or unsatisfiable, for example, P ∧ ¬P.
- A wff is said to be valid if it has value True under all interpretations of its constituent atoms, for example, P ⊃ ¬P.
- Neither valid wffs nor inconsistent wffs tell us anything about the world.

Semantics

- We say that an interpretation satisfies a wff if the wff is assigned the value True under that interpretation.
- An interpretation that satisfies a wff is called a model of that wff.
- An interpretation that satisfies each wff in a set of wffs is called a model of that set.
- The more wffs we have that describe the world, the fewer models there are.
- This means that the more we know about the world, the less uncertainty there is.

Semantics

- Two wffs are said to be equivalent if and only if their truth values are identical under all interpretations. For two equivalent wffs ω₁ and ω₂ we write ω₁ = ω₂.
- If a wff ω has value True under all of those interpretations for which each of the wffs in a set Δ has value True, then we say that
  - Δ logically entails ω
  - ω logically follows from Δ
  - ω is a logical consequence of Δ
  - Δ |= ω
Soundness and Completeness

- If, for any set of wffs $\Delta$ and wff $\omega$, $\Delta \models \omega$ implies $\Delta \models \neg \omega$, we say that the set of inference rules $\mathcal{R}$ is sound.
- If, for any set of wffs $\Delta$ and wff $\omega$, it is the case that whenever $\Delta \models \omega$, there exists a proof of $\omega$ from $\Delta$ using the set of inference rules $\mathcal{R}$, we say that $\mathcal{R}$ is complete.
- When $\mathcal{R}$ is sound and complete, we can determine whether one wff follows from a set of wffs by searching for a proof instead of using a truth table (increased efficiency).

Resolution

- Multiple rules of inference can be combined into one rule, called resolution.
- A clause is a set of literals, which is a short notation for the disjunction of all the literals in the set. For example, the wff $\{P, Q, \neg R\}$ is the same as the wff $P \lor Q \lor \neg R$.
- The empty clause $\{\}$ (or NIL) is equivalent to $F$.
- A conjunction of clauses is a set of clauses that is connected by AND connectives. For example, $\{R, P\}, \{\neg P, Q\}, \{R\}$ stands for $(R \lor P) \land (\neg P \lor Q) \land R$.
- Resolution operates on such conjunctions of clauses.

Resolution

Resolution rule for the propositional calculus:
From clauses $\{\lambda\} \cup \Sigma_1$ and $\{\neg \lambda\} \cup \Sigma_2$ (where $\Sigma_1$ and $\Sigma_2$ are sets of literals and $\lambda$ is an atom), we can infer $\Sigma_1 \cup \Sigma_2$, which is called the resolvent of the two clauses. The atom $\lambda$ is the atom resolved upon, and the process is called resolution.

Examples:
- Resolving $R \lor P$ and $\neg P \lor Q$ yields $R \lor Q$.
  In other words: $\{R, P\}, \{\neg P, Q\}$ yields $\{R, Q\}$.
- Resolving $R$ and $\neg R \lor P$ yields $P$.
  In other words: $\{R\}, \{\neg R, P\}$ yields $\{P\}$.