CS 620 – Theory of Computation – Fall 2009

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Assignment #1

Posted on Sep. 23 – due by Oct. 6 at 7:00pm

Question 1:

Consider the following predicate $P(x, y)$:

$$P(x,y) = \begin{cases} 1 & \text{if} \quad x \geq 2y + 3 \\ 0 & \text{otherwise} \end{cases}$$

a) Write a program in the language $\mathcal{L}$ that computes $P(x, y)$. You cannot use any macros at all.

b) Write down the complete list of snapshots that occur during the computation of $P(5, 2)$. Use the format shown on page 29 in the textbook.

Question 2:

Now you want to write a program in your favorite language $\mathcal{L}$ that computes the square root function $f(x) = \sqrt{x}$. Remember that we only consider natural numbers; for example, $f(49) = 7$, but $f(17) = \uparrow$.

a) Write down the program using macros. You are allowed to use the macros $V \leftarrow V'$, $V \leftarrow m$, GOTO L, and $W \leftarrow f(V_1, \ldots, V_n)$. Use at least one macro of the form $W \leftarrow f(V_1, \ldots, V_n)$, and whenever you do so, provide a program that computes $f(V_1, \ldots, V_n)$.

b) In your program, expand all the macros of the form $W \leftarrow f(V_1, \ldots, V_n)$ using the convention given in Section 2.5 in the textbook. Write down the expanded version of your program, which should now contain no macros except (possibly) those of the forms $V \leftarrow V'$, $V \leftarrow m$, and GOTO L

Question 3:

Give a detailed argument that the following functions are primitive recursive:

a) x^y

b) the predecessor function $p(x)$

These arguments should be of the same form as those provided in Examples 1, 2, and 3 of Section 3.4 in the textbook.

Question 4:

Show that the class of all total functions is a PRC class.

Question 5:

Let $f(x)$ be the sum of the divisors of x if $x \neq 0$; $f(0) = 0$. For example, $f(6) = 1 + 2 + 3 + 6 = 12$. Show that $f(x)$ is primitive recursive.

Question 6:

Let $u(n)$ be the n-th number in order of size which is the sum of two squares. Show that $u(n)$ is primitive recursive.