# CS 620 – Theory of Computation – Fall 2009
# Instructor: Marc Pomplun

# Assignment #2

## Posted on October 8 – due by October 15, 7:00pm

**Question 1:**

a) Write down the code of Program $\mathcal{P}$ in the language $\mathcal{L}$ for which #($\mathcal{P}$) = 1919.

b) What is the number of the following program?

```
[B]   X ← X - 1
      Y ← Y + 1
      Y ← Y + 1
      IF X ≠ 0 GOTO B
```

You do not have to compute the numerical values of expressions such as $3^{27}$ that would result in huge numbers.

**Question 2:**

Do you remember how we used the pairing function and the Gödel numbering to associate each program in the language $\mathcal{L}$ with a unique natural number? To be precise, we demanded that every program in $\mathcal{L}$ is associated with unique number, and we also required that every natural number is associated with a valid program in $\mathcal{L}$. Then we used this numbering to numerically describe the interpretation of programs in $\mathcal{L}$. Now it is your task to develop such one-to-one mappings for other things. If you think that a mapping cannot be defined, please give a reason.

a) Define such a mapping for quintuples of natural numbers.

b) Define such a mapping for pairs (a, b), where a is a natural number and b is a letter from the English alphabet.

c) Define such a mapping for pairs (x, y), where x and y are letters from the English alphabet.

d) Define such a mapping for sequences (of any length) of even numbers.

e) Define such a mapping for strings made up of the letters of the English alphabet, including the empty string, that do not end with the letter x.

Bonus question:

f) Define such a mapping for strings made up of the letters of the English alphabet, including the empty string.

**Question 3:**

Let $H(a, b) = a$ if $\Phi(a, b)\downarrow$
$= \uparrow$ otherwise.

Show that H(a, b) is partially computable.

**Question 4:**

Prove or disprove (you can just give a detailed verbal argument): If there were an upper limit n for the values of all variables in the language $\mathcal{L}$ (so for any variable V, $0 \leq V \leq n$), then HALT(x, y) would be a computable predicate.