

CS 620 – Theory of Computation – Fall 2009

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Assignment #3

Posted on November 13 – due by November 24, 7pm

Question 1: Expanding the Language \mathcal{L} and Causing Trouble

After having worked for some time with the language \mathcal{L} , we start feeling annoyed about the fact that, if we do not want to use macros, there are only three different types of instruction (or four if you count the rather impractical $V \leftarrow V$ instruction type). So we decide to add the instruction type $V \leftarrow 0$ (zero) to our language \mathcal{L} and thereby create a new language \mathcal{L}' . Unfortunately, now we realize that we have to write new universal programs U'_n for the new language \mathcal{L}' as well, and that implies that we also have to devise a new system of enumerating programs in \mathcal{L}' (associating every program with a unique number and every number with a unique program).

Well, anyway, we decided to do this, and now we will stick with this decision. So please

- (a) write down the differences of your new enumerating scheme compared to the one for the original language \mathcal{L} , and
- (b) write down the entire \mathcal{L}' code for the new universal programs U'_n that can execute the code of any \mathcal{L}' program. Of course you can use macros, and you can reuse most of the code that we wrote for programming U_n (see slides and textbook).

Question 2: Prove It!

Let A, B be sets. Prove or disprove:

- (a) If $A \cup B$ is r.e., then A and B are both r.e.
- (b) If $A \subseteq B$ and B is r.e., then A is r.e.

Question 3: Playing around with Strings

Show every step of your calculations for the following questions. Please always complete your calculations; for example, in your answer write 3125 instead of 5^5 .

- (a) Which integer is associated with the string “boston” on the alphabet $A = \{a, b, n, o, s, t\}$?
- (b) What is the string on the alphabet $B = \{c, e, i, k, m, r, s, t\}$ that is associated with the integer 142688?
- (c) Let A and B be alphabets with $A \subseteq B$. Is it true for any natural number n that the string on the alphabet A associated with n is of equal or greater length than the string on the alphabet B associated with n ? Prove or disprove.
- (d) Let n be a natural number and w_A and w_B be the strings associated with n on the alphabets A and B , respectively. Is it true for any n that whenever the length of w_A is greater than or equal to the length of w_B , then $|A| \leq |B|$? Prove or disprove.
- (e) Compute $\text{UPCHANGE}_{2,5}(123)$.
- (f) Compute $\text{DOWNCHANGE}_{3,7}(165)$.
- (g) Compute $\text{LTEND}_3(29)$.
- (h) Given the alphabet $C = \{x, y, z\}$, what is the greatest integer whose associated string on C has a length of four?

Question 4: Far from Unique

Show that for each number u , there are infinitely many different numbers v such that for all x it is true that

$$\Phi_u(x) = \Phi_v(x).$$