Question 1: Expanding the Language $\mathcal{L}$ and Causing Trouble

After having worked for some time with the language $\mathcal{L}$, we start feeling annoyed about the fact that, if we do not want to use macros, there are only three different types of instruction (or four if you count the rather impractical $V \leftarrow V$ instruction type). So we decide to add the instruction type $V \leftarrow 0$ (zero) to our language $\mathcal{L}$ and thereby create a new language $\mathcal{L}'$. Unfortunately, now we realize that we have to write new universal programs $U'_n$ for the new language $\mathcal{L}'$ as well, and that implies that we also have to devise a new system of enumerating programs in $\mathcal{L}'$ (associating every program with a unique number and every number with a unique program).

Well, anyway, we decided to do this, and now we will stick with this decision. So please

(a) write down the differences of your new enumerating scheme compared to the one for the original language $\mathcal{L}$, and

(b) write down the entire $\mathcal{L}'$ code for the new universal programs $U'_n$ that can execute the code of any $\mathcal{L}'$ program. Of course you can use macros, and you can reuse most of the code that we wrote for programming $U_n$ (see slides and textbook).

Question 2: Prove It!

Let $A, B$ be sets. Prove or disprove:

(a) For all sets $A$ and $B$, if $A$ and $B$ are both r.e., then $A \cap B$ is also r.e.

(b) For all sets $A$ and $B$, if $A$ and $B$ are both r.e., then $A \cup B$ is also r.e.

(c) If $A \cup B$ is r.e., then $A$ and $B$ are both r.e.

(d) If $A \supseteq B$ and $B$ is r.e., then $A$ is also r.e.
**Question 3: One More Proof…**

Let $B = \{ f(n) \mid n \in \mathbb{N} \}$, where $f$ is a strictly increasing computable function (i.e., $f(n + 1) > f(n)$ for all $n$). Prove that $B$ is a recursive set.