

CS 620 – Theory of Computation – Fall 2009

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Assignment #4 Sample Solutions

Question 1:

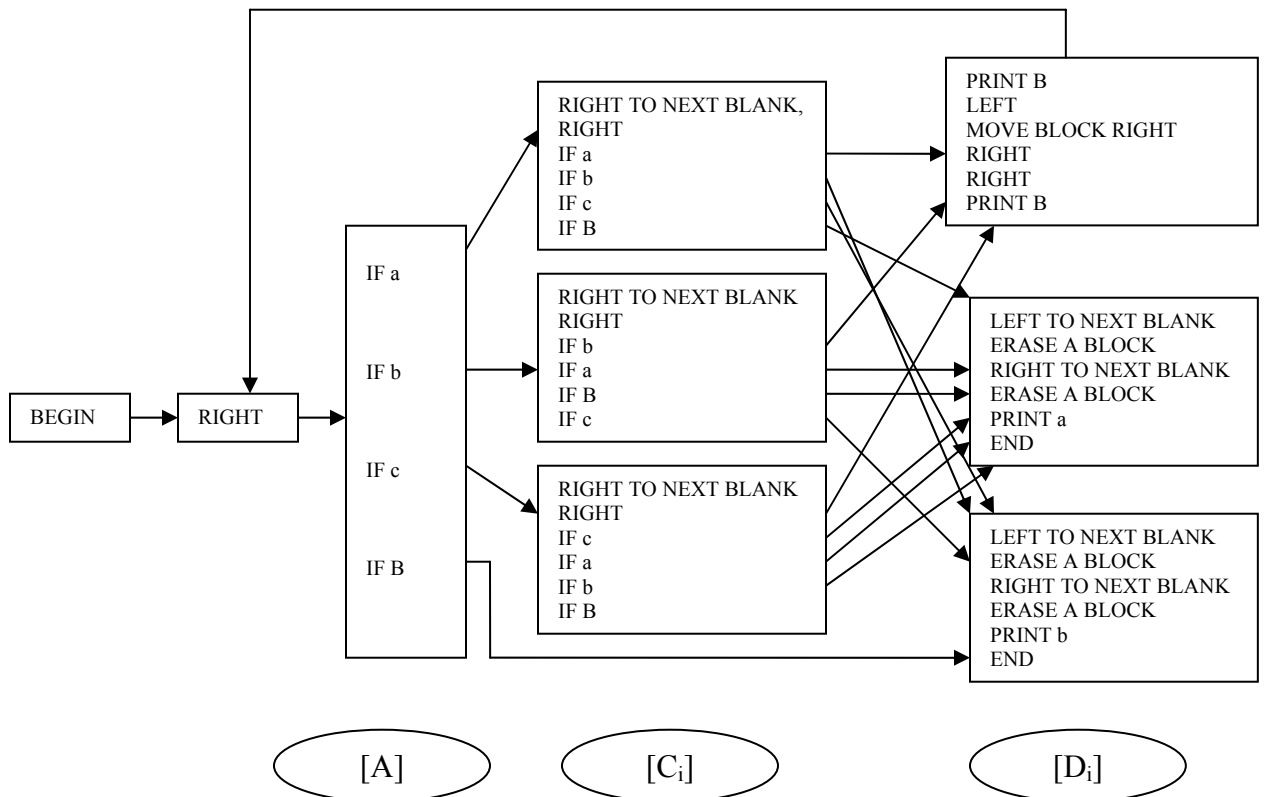
a)

Write a Post-Turing program using an alphabet $A = \{a, b, c\}$ that computes the following function f :

$$f(x, y) = \begin{cases} a, & \text{if } x > y \\ b, & \text{otherwise} \end{cases}$$

Here, $x > y$ means that the string y precedes the string x in alphabetical order. For example, it is true that $ba > abba$, $aab > aaa$, $ccc > cc$, and $bbcb > bbcb$.

You get bonus points if you write a program that computes f strictly.



The macros such as **RIGHT TO NEXT BLANK** are defined in the textbook.

[A] RIGHT
 IF a GOTO C1
 IF b GOTO C2
 IF c GOTO C3
 IF B GOTO D3

[C₁] RIGHT TO NEXT BLANK
 RIGHT
 IF a GOTO D1
 IF b GOTO D3
 IF c GOTO D3
 IF B GOTO D2

[C₂] RIGHT TO NEXT BLANK
 RIGHT
 IF a GOTO D2
 IF b GOTO D1
 IF c GOTO D3
 IF B GOTO D2

[C₃] RIGHT TO NEXT BLANK
 RIGHT
 IF a GOTO D2
 IF b GOTO D2
 IF c GOTO D1
 IF B GOTO D2

[D₁] PRINT B
 LEFT
 MOVE BLOCK RIGHT
 RIGHT
 RIGHT
 PRINT B
 GOTO A

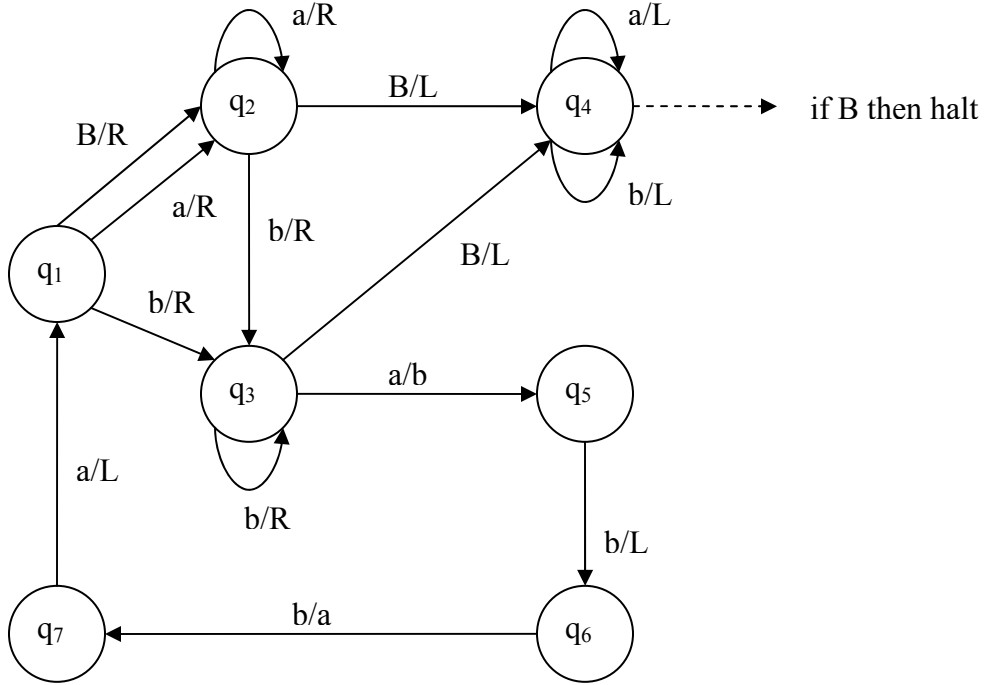
[D₂] LEFT TO NEXT BLANK
 LEFT TO NEXT BLANK
 ERASE A BLOCK
 ERASE A BLOCK
 PRINT a
 LEFT
 GOTO E

[D₃] LEFT TO NEXT BLANK
 LEFT TO NEXT BLANK

↑
B B B B B b a B b b a B
↑
B B B B B b a B b b a B
↑
B B B B B b a B b b a B
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B B B B B b a B B b a B
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Question 2:

Build a Turing machine on the alphabet $A = \{a, b\}$ that computes a function $f(x)$ strictly, where $f(x)$ sorts the symbols in the input string in the order a, b.



Quadruples:

q_1	B	R	q_2
q_1	a	R	q_2
q_1	b	R	q_3
q_2	B	L	q_4
q_2	a	R	q_2
q_2	b	R	q_3
q_3	B	L	q_4
q_3	a	b	q_5
q_3	b	R	q_3
q_4	a	L	q_4
q_4	b	L	q_4
q_5	b	L	q_6
q_6	b	a	q_7
q_7	a	L	q_1

Sequence of configurations:

Baba

↑

q₁

aba

↑

q₂

aba

↑

q₂

aba

↑

q₃

abb

↑

q₅

abb

↑

q₆

aab

↑

q₇

aab

↑

q₁

aab

↑

q₂

aab

↑

q₂

aabB

↑

q₃

aab

↑

q₄

aab

↑

q₄

aab

↑

q₄

Baab

↑

q₄

Machine halts. Output is aab.