

**CS 620 – Theory of Computation – Fall 2009**  
**Instructor: Marc Pomplun**

# Practice Exam

Question 1: \_\_\_\_ out of \_\_\_\_ points

Question 2: \_\_\_\_ out of \_\_\_\_ points

Question 3: \_\_\_\_ out of \_\_\_\_ points

Question 4: \_\_\_\_ out of \_\_\_\_ points

Question 5: \_\_\_\_ out of \_\_\_\_ points

**Total Score:**

**Grade:**

### Question 1: True or False?

Are the following statements true, false, or is their truth value unknown? Check the appropriate box for each statement.

	true	false	unknown
a) A function $f$ is partially computable if and only if there is a program of $\mathcal{L}$ that computes $f$ .	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
b) For all sets $A$ and $B$ , if $A \leq_m B$ and $B$ is recursive, then $A$ is also recursive.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
c) If a non-empty set $S$ is r.e., then $S$ is also the range of a primitive recursive function.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
d) There are infinitely many programs in the language $\mathcal{L}$ that compute the function $f(x) = 2x$ .	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
e) If during a computation by a program $\mathcal{P}$ in the language $\mathcal{L}$ the same state occurs more than once, then this computation will not terminate.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
f) For all sets $A$ and $B$ , if $A \cap B$ is r.e., then $A$ and $B$ are both r.e.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
g) For all sets $A$ and $B$ , if $A \cup B$ is r.e., then $A$ and $B$ are both r.e.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
h) If function $g(x)$ is partially computable and if function $h(x)$ is computable, then function $f(x) = g(h(x))$ is computable.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
i) There is an algorithm that cannot be executed by any program of the language $\mathcal{L}$ .	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
j) For all sets $A$ and $B$ , if $A$ and $B$ are both r.e., then $A \cap B$ is also r.e.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

## Question 2: The Limping Turing Machine

Imagine a Turing machine  $M'$  that works like a normal Turing machine except that whenever it moves to the left, it moves by two squares instead of one. Its movements to the right are by one square as usual. Give a detailed proof of the fact that  $M'$  can compute all functions that normal Turing machines can compute.

**Hint:** Think of the simulation of Turing machines with quintuple Turing machines that we discussed in class.



**Question 3: Programmer's Proof**

Show that if two sets  $A$  and  $B$  are both recursive, then  $A - B$  is also recursive. In your proof, use a program in the language  $\mathcal{L}$  (possibly containing macros).



#### Question 4: A Useful Program

Describe in detail what the following program does. In which case would such a program be especially useful? Why?

```
[A]  IF STP(1)(X, p, T) GOTO C
      IF STP(1)(X, q, T) GOTO E
      T ← T + 1
      GOTO A
[C]  Y ← 1
```



**Question 5: Enumeration**

Explain in detail in your own words why the following theorem is true:

“A set  $B$  is r.e. if and only if there is an  $n$  for which  $B = W_n$ .”