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CS 675
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Assignment 2, Questions 3-7
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Question 3. We know that the equation for any circle of radius, r, centered at point (a, b) is given by $(x - a)^2 + (y - b)^2 = r^2$. We have three valid pairs of values for x and y, so we can write three equations and solve for the three unknowns, a, b, and r. First we write the original equations, then expand them.

(1) $(1 - a)^2 + (1 - b)^2 = r^2 \quad \Rightarrow \quad 1 - 2a + a^2 + 1 - 2b + b^2 = r^2$

(2) $(-2 - a)^2 + (4 - b)^2 = r^2 \quad \Rightarrow \quad 4 + 4a - a^2 + 16 - 8b + b^2 = r^2$

(3) $(0 - a)^2 + (6 - b)^2 = r^2 \quad \Rightarrow \quad a^2 + 36 - 12b + b^2 = r^2$

Simplifying the three equations on the right yields:

(1a) $a^2 - 2a + b^2 - 2b + 2 = r^2$

(2a) $a^2 + 4a + b^2 - 8b +20 = r^2$

(3a) $a^2 + b^2 - 12b + 36 = r^2$

Subtracting (3a) from (2a) yields:

$$4a + 4b - 16 = 0 \quad \Rightarrow \quad 4a + 4b = 16 \quad \Rightarrow \quad 4(a + b) = 16 \quad \Rightarrow \quad a + b = 4$$

which finally yields:

(4) $b = 4 - a$

Subtracting (1a) from (3a) yields

(5) $2a - 10b + 34 = 0$

Substituting (4) into (5) gives

$$2a - 10(4 - a) + 34 = 0 \quad \Rightarrow \quad 2a - 40 + 10a + 34 = 0 \quad \Rightarrow \quad 12a - 6 = 0 \quad \Rightarrow \quad 12a = 6$$

yielding

(6) $a = 0.5$

Plugging (6) back into (4) yields $b = 4 - 0.5$, or

(7) $b = 3.5$

Finally, to find r, or better yet, r^2, we can use any of the three original equations; *e.g.*, (3).

$$(0 - 0.5)^2 + (6 - 3.5)^2 = r^2 \quad \Rightarrow \quad (-0.5)^2 + 2.5^2 = r^2 \quad \Rightarrow \quad 0.25 + 6.25 = r^2$$

or

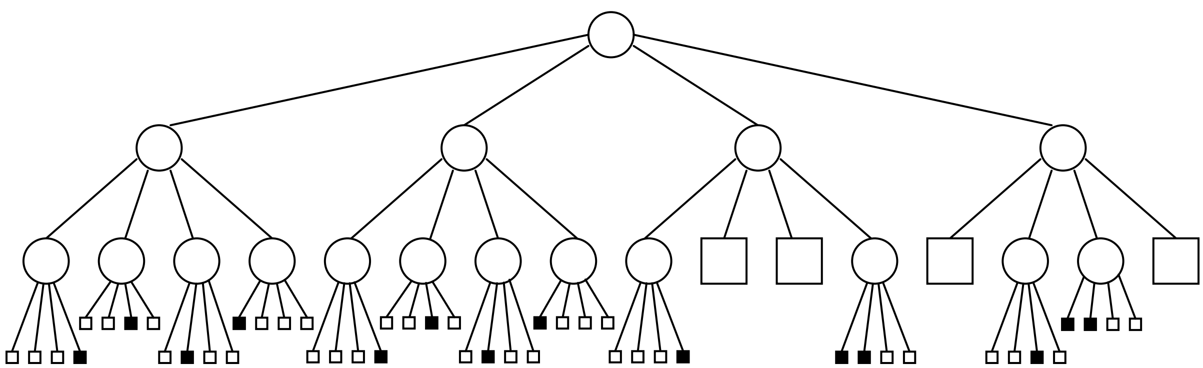
(8) $r^2 = 6.5$

Now we can write the equation to find any point on this circle,

(9) $(x - 0.5)^2 + (y - 3.5)^2 = 6.5$

The explicit answer to the question is that the center of the circle is at (0.5, 3.5) and the radius is $(6.5)^{½}$, which is ~2.55. If you substitute the three supplied values of x and y into the left side of (9), it will indeed yield 6.5. This is left to the reader as an exercise.

Question 4.



Question 5(a). The median filter alternates between having nine 0s and nine 150s. The result is

0	150	0
0	150	0
0	150	0

The averaging filter has nine values of 1/9. The result is

50	100	50
50	100	50
50	100	50

Question 5(b). To blur the image, you should use the averaging filter. The median filter excels at removing extreme values; the filtered pixel value is determined by how common various values are in the filtered area. It does not necessarily do anything to the contrast between two adjacent regions, however. In the example above, the median filter just shifted a line; it didn't blur anything. The averaging filter, on the other hand, tends to change all of the values in the vicinity of a pixel and changes them to the average of those values. This is going to make all larger-than-average values smaller, and all smaller-than-average values larger. That brings all pixels closer to the (local) center, which reduces contrast; *i.e.*, causes blurring.

Question 6(a). The four co-occurrence matrices for (1, 1) are

8	0
0	17

Texture: (1)

8	5
5	7

(2)

0	10
15	0

(3)

12	0
0	13

(4)

The four for (1, –1) are

0	9
8	8

Texture: (1)

9	5
3	8

(2)

0	10
15	0

(3)

13	0
0	12

(4)

Question 6(b). My solution for distinguishing textures is to take the absolute value of each of the four differences between each co-occurrence matrix value for the (1, 1) co-occurrence matrices, then average the four of them. Do the same thing for the (1, –1) co-occurrence matrices, then average the values of the two different sets. For example, Textures (1) and (2) have differences of 0, 5, 5, and 10, which averages to 5, for (1, 1). Similarly, the differences are 9, 4, 5, and 0 for (1, –1), which averages to 4.5. The average of 5 and 4.5 is 4.75, which is entered in the table below for (1, 2) and (2, 1).

Texture	(1)	(2)	(3)	(4)
(1)	0	4.75	8.25	5.25
(2)	4.75	0	8	4.5
(3)	8.25	8	0	12.5
(4)	5.25	4.5	12.5	0

Looking at the table, the most similar textures are (2, 4), with (1, 2) very, perhaps insignificantly, close. (1, 4) are also similar, while (2, 3) and (1, 3) are noticeably less similar, and (3, 4) the least similar. Looking at the patterns with the naked eye generally confirms these results; (3, 4) definitely looks least similar, while one can argue about (1, 2) vs. (2, 4), or even (1, 4), all of which are within 1 of each other. Larger co-occurrence matrices should improve the results.

Question 7. According to my old calculus textbook (*Calculus and Analytic Geometry, Fifth Edition*, by Thomas and Finney), which I purchased during the *Carter* Administration, the formula for finding the radius of an osculating circle is

$$r = \frac{(1 + f'(x)^2)^{3/2}}{\left| f''(x) \right|}$$

Given our curve of $f(x) = x^2$, $f'(x) = 2x$ and $f''(x) = 2$. For the point (1, 1), $f'(x) = 2$; while $f''(x)$ is always going to equal 2. Plugging the numbers into the formula yields a radius of

$$\frac{5^{3/2}}{2} \text{ or } \frac{5\sqrt{5}}{2} \text{ or } \sim 5.6.$$

For the point (2, 4), $f'(x) = 4$, and $f''(x)$ is still 2, of course. The result is

$$\frac{17^{3/2}}{2} \text{ or } \frac{17\sqrt{17}}{2} \text{ or } \sim 35.$$