When More Seems Less – Non-Spatial Clustering in Numerosity Estimation

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Abstract

How is numerosity estimation affected by additional structural information in visual displays? Two experiments investigated if the linking of dots by line segments, thereby forming clusters of polygons, leads to an underestimation effect similar to that observed in classical experiments on clustering by spatial proximity. Our findings confirmed such an underestimation effect for non-spatial clustering. In addition, the relative magnitude of the underestimation increased along with the number of objects and with cluster size. Finally, we observed that the presence of an equivalent number of line segments as in the previous condition, however unaligned with the dots, reduced the underestimation effect to a constant relative magnitude.

1 Introduction

Various factors seem to influence the accuracy of *numerosity estimation*. Hamilton (1859), Miller (1956) and Atkinson et al. (1976) established the *number* of objects to be one of these factors. They showed that observers could grasp a certain number of objects at a glance, but differed in the extent up to which such “subitizing” is possible. The values given ranged from 4 to 7 objects. The *size* of the objects also affects estimation accuracy. Binet (1890), Messenger (1903) and Ginsburg and Nicholls (1988) showed that the number of objects is overestimated with increasing object size. Another factor is *spatial proximity* of objects. Piaget (1965) and Krueger (1972) reported that the estimated number of objects increases when their spacing is increased.

Several studies investigated the effect of object *arrangement* on numerosity estimation. Frith and Frith (1972) discovered the so-called “solitaire illusion”: A display seems to show more objects when they are grouped into few large *clusters* than when they form many small ones. Messenger (1903) was the first to point out that a regular pattern of objects seems to comprise more dots than an irregular one, a phenomenon termed “regular-random illusion” by Ginsburg (1980). Subsequent studies showed that clustered objects are underestimated to an even larger degree than randomly arranged ones. Accordingly, the regular and random arrangements are only two locations on a continuum which ranges from regular over random to clustered (Watler, 1984; Ginsburg & Goldstein, 1987; Ginsburg, 1991). In these previous studies the clustering effect depended on spatial proximity since strong clustering led to a decrease in average distance within clusters.

In the present studies, we considered a factor complementary to spatial proximity and arrangement, namely the *impact of additional structural information* on numerosity estimation. Our experiments investigated if the clustering effect (underestimation) persists when clustering is achieved by line segments that combine single dots into *clusters of polygons*, i.e. when a *non-spatial* clustering mechanism is used. At first sight, this hypothesis seems implausible as the line segments add to the number of elements in the display. If perceived numerosity is a monotonously increasing function of the overall number of objects, the adding of line segments should rather lead to an overestimation. In addition, we investigated if cluster size (i.e. number of objects in a cluster) affects the estimation as well. According to the solitaire illusion, few large clusters seem to comprise more objects than many small ones. The underestimation should thus be more pronounced for small clusters.

Experiment N1 investigated if the expected effect on perceived numerosity can be observed at all and whether this perceptual phenomenon is influenced by cluster size and by the number of objects. The aim of Experiment N2 was to establish whether the clustering function of the polygons in Experiment N1 is a prerequisite for the underestimation or if the mere presence of the polygons is sufficient for the effect.

2 Experiment N1 – Numerosity in the Presence of Clustering Polygons

We investigated the effects of object clustering by line segments on perceived numerosity, choosing an experimental method that differed from those used in most of the previous experiments on numerosity estimation. Participants did not state the number of objects. Instead, they adjusted a variable set of dots (comparison stimulus) to match (in numerosity) a given set of dots (target stimulus). In the experiment, we compared the estimation accuracy between unclustered sets, “small” and “large” clusters. We used various numbers of dots.

2.1 Method

The participants were 25 experimentally naive students. The stimulus displays consisted of two hemifields: The target stimulus was presented in the left hemifield while the comparison stimulus was shown in the right hemifield. Objects in both hemifields were dots with a diameter of 0.18^o. The dots in the target stimulus were linked by line segments (line segment width: 0.04^o) so as to form polygons. These constituted clusters of equal size within the same stimulus display. Dots remained clearly visible. Both dots and polygons were shown in fully saturated red on a black background.

The participants used computer-mouse movements to adjust the number of dots in the comparison stimulus so as to subjectively match the number of dots in the target stimulus. The stimulus set consisted of 196 different distributions of dots and was generated by combining 7 numbers n_i of dots ($n_i = 10+20 \cdot i,\ i = 1, \ldots, 7$) with 3 cluster size levels (*unclustered; small clusters* – 2, 3 or 4 dots per cluster; *large clusters* – 10, 20 or 30 dots per cluster). The 7 displays from the unclustered control condition were shown 4 times, those from the two clustered conditions 12 times each. The comparison stimulus consisted of a random number of dots between 1 and 300 (before adjustment). In both the target and comparison stimuli, dots were randomly positioned and equally distributed, keeping a minimum distance of 0.4^o between centers of neighbouring dots (see Figure 1 (left)).

Every participant was presented with the entire set of stimuli in random order. No time limit was imposed, however participants were instructed “to adjust the number of items as quickly and accurately as possible”.

2.2 Results and Discussion

For the statistical analysis we computed the relative estimation error as percentages. Negative values indicated an underestimation of the number of dots in the target stimulus, positive values indicated an overestimation, accordingly. Figure 1 (right) shows the mean estimation errors for the factors cluster size and number of dots.

On average, the estimation error was -24%, which differed significantly from 0 $[F(1, 24) = 264.34; p < 0.001]$. An analysis of variance led to significant main effects for cluster size $[F(2, 48) = 79.87; p < 0.001]$ and number of dots $[F(6, 144) = 18.10; p < 0.001]$. The interaction between these two factors was also significant $[F(12, 288) = 14.06; p < 0.001]$.

Post-hoc tests of the main effect of cluster size showed that the mean for the unclustered control condition did not differ significantly from 0 (-4.4%). As expected, however, it differed significantly from the two clustered conditions (-30.0% for small clusters and -37.4% for large clusters). In addition, the two clustered conditions differed from each other: On average, large clusters led to a stronger underestimation than small ones [Scheffé : $\mathit{Diff}_{crit} = 6.15; p < 0.05$]. We also conducted post-hoc tests for the main effect of number of dots. All means for dot numbers n_i, n_j with $|i - j| > 1$ (i.e. not directly neighboured levels of the factor number of dots) differed from each other. Exception: The pairing $(n_5,\ n_7)$ [Scheffé : $\mathit{Diff}_{crit} = 5.81; p < 0.05$].

The interaction of cluster size and number of dots was an ordinal one: Means remained relatively constant across the numbers of dots in the unclustered condition. In contrast, the underestimation increased along with the number of dots for both clustered conditions. Moreover, the underestimation for the control condition was smaller than that of the clustered conditions for any number of dots. Finally, the two clustered conditions differed significantly for few dots (30 and 50), but estimation errors converged with an increasing number of dots. The estimation error in the small cluster condition started with lower underestimation values and increased more rapidly than the values in the large cluster condition [Scheffé : $\mathit{Diff}_{crit} = 9.26; p < 0.05$].

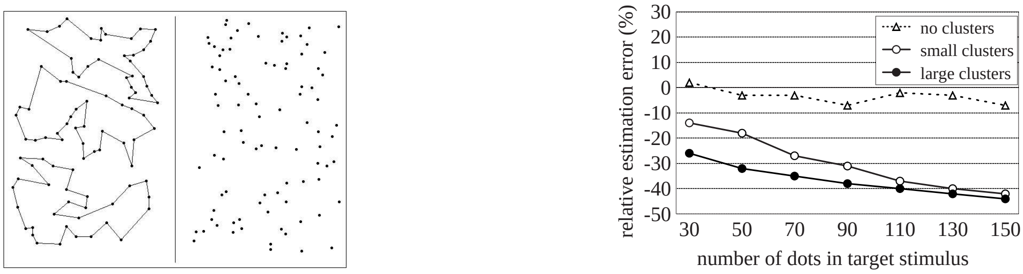


Figure 1: Left: Sample stimulus used in Experiments N1 (cluster size 30, 90 dots in target stimulus). Right: Mean relative estimation errors in Experiments N1 by cluster size and number of dots in target stimulus.

In summary, Experiment N1 has shown that *clustering* of objects by additional structural information does lead to a pronounced *underestimation* of the number of objects. This finding is in line with the effects found for clustering by spatial proximity. However, in contrast to the observations on spatial clustering we could *not* establish a decrease in the magnitude of this underestimation effect with increasing cluster size. Instead, the opposite was found: The underestimation increased (at least with small numbers of dots). In addition, the underestimation also increased with the number of dots.

3 Experiment N2 – Numerosity in the Presence of Non-Clustering Polygons

The second experiment investigated if the linking of dots is a prerequisite for the underestimation effect – as suggested by Experiment N1 – or if the mere presence of the line segments is already sufficient. Experiment N2 differed from Experiment N1 in that the line segment links that had previously clustered the dots were no longer aligned with the dots. If clustering causes the underestimation this manipulation should eliminate the effect.

3.1 Method

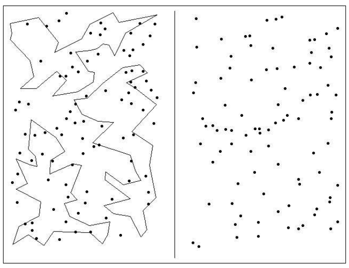
The participants were 10 experimentally naive students. Apparatus, stimuli and procedure were the same as in Experiment N1. However, instead of linking dots so as to form polygons, the polygon junctions were not directly placed on dots. Instead, polygons were randomly positioned among the dots, retaining their shape and their closed form from Experiment N1. Again, all polygons in a target stimulus had the same size, the total number of polygon junctions being equal to the number of dots in the target stimulus. Since the polygon edges do not link the dots in Experiment N2 we shall use the term "polygon size" in the following, denoting the number of polygon junctions (see Figure 2 (left)).

3.2 Results and Discussion

As in the previous experiment, we calculated the relative estimation error for the factors polygon size and number of dots. Figure 2 (right) shows the results for both factors.

The mean underestimation in Experiment N2 was -12.95%. This value still differed significantly from 0 $[F(1,9) = 17.43; p < 0.01]$. Obviously, the underestimation did *not* disappear in Experiment N2. The analysis of variance revealed a significant main effect only of polygon size $[F(2,18) = 12.88; p < 0.001]$. Post-hoc tests showed that again the mean of -3,80% for the "unclustered" condition was not significantly different from 0, but differed from the two "clustered" ones (-16.26% for small polygons and -18.78% for large polygons). The two "clustered" conditions did not differ from each other $[\text{Scheffé} : \textit{Diff}_{crit} = 8.42; p < 0.05]$.

The comparison of the results from Experiments N1 and N2 yielded a significant main effect of the factor "experiment" $[F(1,33) = 11.03; p < 0.01]$: The underestimation effect was *less pronounced* in Experiment N2. In addition, it is interesting to note that the effect of cluster/polygon size that was present in Experiment N1 but absent in Experiment N2 led to a significant interaction between the factors experiment and cluster/polygon size $[F(2,66) = 16.73; p < 0.001]$. Subsequent post-hoc tests showed that in Experiment N1 all means of cluster sizes differed from each other. In contrast, only the respective means of the "clustered" conditions were different from that of the "unclustered" control condition in Experiment N2. Furthermore, only the clustered conditions but not the unclustered condition showed a difference between the experiments: The values in Experiment N2 were lower than in Experiment N1 for both cluster/polygon sizes. The main effect of the number of dots, which was only present in Experiment N1, resulted in a significant interaction between the factors experiment and number of dots $[F(6,198) = 3.94; p < 0.01]$. Likewise, the interaction between cluster/polygon size and number of dots that was only present in Experiment N1 yielded a significant 3-way interaction between experiment, cluster/polygon size and number of dots $[F(12,396) = 3.75; p < 0.01]$. In addition, the main effects for cluster/polygon size $[F(2,66) = 37.04; p < 0.001]$, the number of dots $[F(6,198) = 6.48; p < 0.001]$ and the interaction between cluster/polygon size and number of dots $[F(12,396) = 4.61; p < 0.001]$ also reached significance.



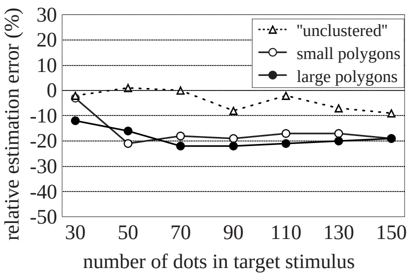


Figure 2: Left: Sample stimulus used in Experiments N2 (cluster size 30, 90 dots in target stimulus). Right: Mean relative estimation errors in Experiments N2 by polygon size and number of dots in target stimulus.

In summary, the *random arrangement* of the polygons significantly *reduced the underestimation*; it also eliminated the effect of the number of dots and the difference between the two clustered conditions. Apparently, the mere presence of the additional polygons leads to a *constant underestimation*.

4 General Discussion

We observed a significant *underestimation* of objects when the objects were clustered by line segments into polygons in Experiment N1. This result showed that in general, additional structural information leads to errors in numerosity estimation that resemble those found when clustering was achieved by spatial proximity. However, this finding must be considered a *paradox* in view of the non-spatial clustering mechanism that we used: Despite adding line segments and therefore increasing the overall number of objects in the target stimulus, perceived numerosity was not increased. Further results showed that, contrary to our expectations, underestimation increased for larger cluster size instead of the predicted decrease. In addition, the underestimation increased with the number of objects.

The question remains how we could explain these results. One possible interpretation for our findings is compatible with Brunswik's (1934) classical view. He argued that numerosity estimation is a compromise between the conscious intention to assess the number of dots and the latent intention to judge the number of clusters. He termed this view "poles of intentions". This view could be restated in modern terms and transferred to our investigation. The interpretation then suggests that the number of clusters cannot be ignored when estimating the number of dots. One reason for this incapability might be the fact that the line segments were superimposed on the dots. If we assume that the observer computes the number of dots as a type of weighted mean of dots and objects, the effect of cluster size can be explained: The stimulus displays either few large clusters or many small ones. The overall number of perceived objects is less for large clusters than for small ones, consequently leading to a higher weighted mean for the large clusters. Assuming that the influence (weights) of these two factors is variable, the increasing underestimation along with the number of dots may also be explicable. With an increasing number of dots, participants apparently find it more and more difficult to concentrate on the assessment of dots, but rather consider clusters. This leads to corresponding adaptations of the weights – the increase of that for the "cluster pole" and the according decrease of the "dot pole".

The results of Experiment N2 demonstrated that the *mere presence of polygons leads to an underestimation*, which is independent of cluster size and number of dots. Although polygons were randomly arranged, they still enclosed a certain number of dots. If we assume that the polygons "mask" the enclosed dots, the observed underestimation may be explicable. An increasing number of dots leads to a proportional increase in the number of clusters, and therefore a constant proportion of dots is enclosed by the polygons. In analogy, the probability that objects are enclosed by a polygon is similar, independent of whether polygons consist of many small clusters or of some large ones.

The observed effects of non-spatial clustering mechanisms revealed that the apparently simple assessment of numerosity is already a rather complex perceptual capability. It deserves more attention than it has received during recent years, particularly in the light of the significant importance that rapid and reliable numerosity estimation can have in many situations in daily life. Further research is currently under way. Our new studies, integrating data from eye-movement recordings, are aimed at validating the current findings and investigate the effects of alternative clustering methods on perceived numerosity. We also attempt to implement a computational model based on artificial receptive fields in order to assess the correctness of our explanatory approaches suggested for the empirical observations.

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