1. Exercise 2.1 in the handout.
2. Exercise 2.2 in the handout.
3. Exercise 3.2 in the handout.
4. Exercise 5.1 in the handout. (I’m not expecting something long here. But it has to be convincing.)
5. Let $G = (V, E)$ be a bipartite graph. Let’s assume the two “parts” are $v_1$ and $V_2$, so $v_1$ and $V_2$ are disjoint and $V_1 \cup V_2 = V$. And we know that all edges go from a node in $V_1$ to a node in $V_2$.

A graph is said to be $k$-regular (for some $k \geq 1$) iff each vertex has exactly $k$ edges incident on it.

We say that a bipartite graph (as above) has a perfect matching iff there is a set of edges $E_0$ such that

- Each edge joins a vertex in $V_1$ to $V_2$. (Well, we know this is automatically true in any case, right?)
- Each vertex in $V_1$ is the endpoint of exactly one of the edges in $E_0$.
- Each vertex in $V_2$ is the endpoint of exactly one of the edges in $E_0$.

Prove that if there is some $k \geq 1$ for which $G$ is $k$-regular, then $G$ has a perfect matching.