CS310 – Advanced Algorithms and Data Structures

Spring 2021

More on Sets
Sets

- Consider set of integers: $A = \{1, 5, 3, 96\}$, or $B = \{17, 5, 1, 96\}$ and a set of strings, \{"Mary", "contrary", "quite"\}.

- Sets have no duplicates and order doesn't count. (vs. List allows duplicates and order does count.)

- The Set API is just the Collection API.

- So Sets have methods isEmpty, size, add, contains, remove, clear, toArray, iterator.

- We will see the Collection API in fact has more methods, but first look at basic OO considerations.
How does a Set hold its element objects?

Set of Integer objects: Each number here is in an Integer object, hanging off the set object by its ref:

Ref to HashSet object:

```
a = {1, 3, 5, 96}
```
How does a Set hold its element objects?

Each element here is in an Integer object, with a ref that can be used in multiple places ("aliasing"): 

Ref from variable:

Integer \( x = 5 \);
\( a \).add(\( x \));

\( a = \{1,3,5,96\} \)
How does a Set hold its element objects?

In general, we may be able to *change the element inside the set* by using this “extra” ref, but in this case Integers are *immutable*, so with Integer (or String) elements this can’t actually happen.

Ref from variable:
Integer x = 5;
a.add(x);

\[ a = \{1, 3, 5, 96\} \]
How does a Set hold its element objects?

We programmers need to keep in mind that our Set elements can be shared. The encapsulation of Sets covers their handling and storage of the their element refs, but not the element objects themselves.

Ref from variable:
Integer x = 5;
a.add(x);

a={1,3,5,96}

P.S. Same is true of elements of Lists, Maps, etc.
How does a Set hold its element objects?

We programmers need to keep in mind that our Collection elements can be shared, and mutated while in the Collection

Ref from variable:
BankAccount ba1 = new … ;
a.add(ba1);
ba1.withdraw(100)

A BankAccount object is mutable.
More Collection/Set operations

• The Collection interface we’ve been using contains just the basic set operations add, contains, remove, etc.

• We would like to use Set Union, Intersection, and Difference.

• There are additional Collection methods that allow us to do these operations…
Basic set operations - reminder

\[ A \cup B = \{1, 3, 5, 17, 96\} \]

\[ A \cap B = \{1, 3, 5\} \]

\[ A - B = \{3\}, \quad B - A = \{17\} \]

From Wikipedia
Set operation examples: union

- Consider Sets made with HashSet:
  ```java
  Set<String> a = new HashSet<String>();
  ```

- Union of Set<String> a and Set<String> b: we can “addAll” of b to a or vice versa, to form the union set:
  ```java
  a.addAll(b)  // a is the union of a and b
  b.addAll(a)  // turns b into the union
  ```

- If we don’t want to change a or b, we make a new set from one of them first:
  ```java
  Set<String> union = new HashSet<String>(a);
  union.addAll(b);  // form a UNION b in own set
  ```

- Here we are using HashSet<E>’s second-listed constructor `HashSet(Collection<? extends E> c)`
  - which promises to make a HashSet out of any Collection<E>, and even Collection <F> where F is a subtype of E, so an F ISA E.
Intersection example

```java
a.retainAll(b);
// turns a into intersection of a and b
b.retainAll(a);
// turns b into intersection
Set intersection = new HashSet<T>(a);
intersection.retainAll(b);
// form a INTERSECT b in own set
```
Subset Containment example

- For subset containment, we just use `containsAll`: The following returns true if `b` is a subset of `a`:

  ```java
  a.containsAll(b);
  ```
Difference example

Notice that $A - B$ and $B - A$ are different.

```java
a.removeAll(b); // turns a into a - b
b.removeAll(a); // turns b into b - a
Set<T> diffAB = new HashSet<T>(a);
// form a - b in own set:
diffAB.removeAll(b);
// form b - a in own set
    Set<T> diffBA = new HashSet<T>(b);
diffBA.removeAll(a);
```

Note: $(a - b) \cup (b - a)$ measures how different sets $a$ and $b$ are.
Implementation of union, intersection

**UNION via** `a.addAll(b)`:  
- Iterate through `b`, checking elements for membership “contains()” in `a`, an $O(1)$ op for HashMap, $O(\log n)$ for TreeMap. Any that aren’t there, add (by ref copy, as usual) to result set, also $O(1)/O(\log n)$. For $N$ elements of `B`, $O(N)$ operation for HashMap, $O(N\log N)$ for TreeMap.

**INTERSECTION via** `a.retainAll(b)`:  
- Iterate through `a`, checking elements for membership in `b`. If not in `b`, remove it ($O(1)/O(\log n)$). For $N$ elements of `A`, an $O(N)$ operation for HashMap, $O(N\log N)$ for TreeMap.
Implementation of difference

• For `a.removeAll(b)` it is possible to be a little smarter, as is done in the JDK code...

```java
if (a.size() < b.size())
    iterate through a, removing (from a) those that are in b
else
    iterate through b, removing those from a
```

• This way, only the smaller set is scanned.

• Thus, when we call `a.removeAll(b)`, we know it only scans the smaller set.
How does it look with the element objects?

- In Java we allow many ref’s to one object, and when we call “new HashSet<E>(a)” we are not copying the element objects but just their refs, a “shallow” copy.
- Then the addAll copies more element refs to the new set.
- Element matching is done via element.equals()
- And if we’re using HashSet, we need a consistent hashCode for the elements as well. For TreeSet, we need a consistent compareTo.
How does it look with the element objects?

Disjoint sets and their union:

\[ A = \{1, 5\} \]

\[ B = \{17, 96\} \]

\[ A \cup B = \{1, 5, 17, 96\} \]
How does it look with the element objects?

Sets with shared elements and their union:

$A = \{1, 5\}$

$B = \{1, 17, 96\}$

$A \cup B = \{1, 5, 17, 96\}$
How does it look with the element objects?

Sets with equal but not shared elements and their union:

\[ A = \{1, 5\} \]

\[ B = \{1, 17, 96\} \]

\[ A \cup B = \{1, 5, 17, 96\} \]

Equal: \( e1.equals(e2) \) is true
Toy example revisited: vowels

- The original pseudocode said
  
  ```
  if (c is a vowel)
  count it
  ```

- How can we code the condition here?

  ```
  String vowelStr = "aeiou";
  Set<Character> vowels = new HashSet<Character>();
  for (int i=0;i<vowelStr.length(); i++)
    vowels.add(new Character(vowelStr.charAt(i)));
  ```

- Later: // c a char: O(1) lookup

  ```
  if (vowels.contains(c))
  count++;
  ```
Toy example revisited: vowels

- Of course for 5 items this is not important, but for large sets, it really matters.
- We have gone from an O(N) search to an O(1) lookup, a big savings.
- So when you see a loop of tests, think “could I use a Set here?”
- We just saw how set-thinking allowed us to replace an O(N) search with an O(1) lookup when we needed to classify something.
- But to see the full pay-off of set-thinking we need to use the set-crunching operations of union, intersection, etc.
Example: monitoring users

• Suppose we have 500 office connections, or “lines”, each line has a Set of usernames seen on the line, so this defines 500 Sets of usernames.
• This is like the problem setup in project 0.
• For example, the set of usernames (Strings) seen on line 1, set of users seen on line 2, etc.
• We could hold this all in an ArrayList<Set<String>> if we wanted.
• Or in an array of Set<String>, using the trick of casting from raw type to generic type:

```java
Set<String> lines[] =
    (Set<String>[])(new HashSet[500]);
```
Example: monitoring users

• We have 500 sets of String usernames: users on line 1, users on line 2, etc.

• How do you find all users ever seen on any line?
  ✓ Answer: find the union of all the sets.

• How do you find if anyone (or several users) logged in on all lines?

• Suppose one line is for a public station suspected of break-in attempts. The set of users who logged in on that line are considered suspicious. How do you find all the lines that any of these users logged in on?
Application idea – movie rentals

- Netflix and other webapps offer movies for rent. Thinking of new features…
- Suppose we get our patrons to rate movies they have just seen: buttons for "liked movie" and "disliked movie".
- Then our webapp could enter this data into the customer-movie database. Movies have well-defined identifiers, movie titles, or title and year if needed.
- We want our webapp to output suggestions, based on our like-dislike data from our own customers, or choose the next movie for binge-watching.
- These suggestions are predictions of liking a certain movie Y, given knowledge of previous movie likes X.
Application – movie rentals

• After a while we would have access to the movie like-sets and dislike sets of our customers.
• This is now happening in a lot of industries, and is summarized by the phrase "market of one" data.
• Looking from the movie angle, we have sets of customers who liked it and disliked it.
• The simplest analysis is simple movie popularity:
  \[ \text{popularity} = \frac{\text{Size(likeset)}}{\text{Size(likeset)} + \text{Size(dislikeset)}} \].
Application – movie rentals

• But what about these predictions from X to Y?
• In the population, some people liked both X and Y, some liked X only and some liked Y only and some disliked both. If the first and last of these are relatively large compared to the middle ones, we have correlation between them: they tend to go together, like or dislike.
• But we don't actually care about the dislike-X case for our predictions. We are interested in all the cases where people liked X—then what about Y? The chance that such a person will like Y is (statistically)
  - probability(like Y | like X) = Size(like(X) ∩ like(Y)) / Size(like(X))
Application – movie rentals

- After a while our customers think we're great and really come to depend on our predictions. Then they become more predictable themselves.

- Although it is impossible to keep enough movies on hand to keep everything available, we now have a notion of a good substitute:
  - if X predicts Y, then Y is a good substitute for X. Each movie has a substitute list based on some predictability level we've established.
How do we implement this?

• Each movie has a substitute set. movie → { substitutes } i.e., a Map.

• We also have movie status, whether or not it’s available. We can combine these into one master Map, using notation (x,y) for an object with fields x,y:

  movie → (status, {substitutes})

• Here the domain element is a movie, id’d by String movie title. The range element is an object with an Boolean status and a Set of movies, i.e., Set of Strings.

• Pretty easy to work with since String comes with equals and hashCode, ready for HashMap.

• Implementation: HashMap<String, MovieData> where MovieData is a class which has a Set<String> as well as a status (true = available).
Program: Counting "gaps" in our movie collection

- If a movie isn’t available, and all its substitutes are also all rented out, let's call it a "gap" in our collection.
- Given the master map M, count the gaps in the movie inventory.
- First try at an algorithm for this follows.
FYI: Implementation: first attempt

// Counting "gaps" in our movie collection: we
// don't have the movie or any of its substitutes.

int gapCount = 0;
loop through movies in M, a Map of String to (status, {subs})
movieData = M.get(movie); // look up movie in Map
if (movie’s status == false) {
    loop through movie’s subs, Set of Strings {
        // look up sub’s status via M
        M.get(sub) yields sub’s status
        if (sub’s status == true)
            // found substitute, stop searching
            break;
    }
    if (no sub found) gapCount++;
}
set-oriented implementation

- Note that in the above code we're searching through a set—that's often avoidable.
- We are trying to determine availability—we can make a Set of available movies and use that, or maintain these sets in the database along with the other data that supports.
- Here's an algorithm that first computes these availability sets and then uses them in the main computation.
FYI: Set oriented implementation

// Counting "gaps" in our movie collection: version 2 using more set-thinking

// first compute avail and unavail movie sets
loop through movies in M, a Map of String to (status, {subs})
  movieData = M.get(movie); // look up movie in Map
  if (movieData’s status = true)
    avail.add(movie); // compute set of available movies
  else
    unavail.add(movie); // and unavailable ones
// second, check out unavail movies:
// look at (their subs) intersect (avail movies)
int gapCount = 0;
loop through unavail Set of movie title strings
  movieData = M.get(movie); // get subs
  availSubs = subs INTERSECT avail
  if (availSubs.isEmpty())
    gapCount++
We need to use retainAll to do the intersection, but retainAll modifies its own object, so we need a new Set to use as a temporary, throw-away Set here:

```java
// calculate new set = intersection of subs and avail
Set<String> availSubs = new Set<String>(subs);
availSubs.retainAll(avail); // intersect subs and avail
if (availSubs.isEmpty()) // if none of the subs are avail
    gapCount++;
```

We have not yet done the full performance analysis to see if these examples in fact save time this way.

Sometimes set crunching does pay off generously in applications.

Also, some things are greatly simplified by this approach.
We have been mostly using HashMap for Maps so far, but sometimes TreeMap is a better fit.

Both HashMap and TreeMap implement the Map interface.

HashMap has slightly faster lookup (get), O(1) vs. O(log N) for TreeMap, but log(N) is very good.

The TreeMap (and TreeSet) maintains its keys in an order, whereas hashing randomizes the keys.

Thus we can avoid a sort by using a TreeMap in some cases.
Spell Checker Example

- We can read a file of words into a Set<String> and use it to check spelling of words from a text.
- File of words on UNIX/Linux /usr/share/dict/words
- Bits of it: see it includes variations, not just the root word:

<table>
<thead>
<tr>
<th>Americanism</th>
<th>admirable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Americanism's</td>
<td>admirably</td>
</tr>
<tr>
<td>Americanisms</td>
<td>admiral</td>
</tr>
<tr>
<td>Americanization</td>
<td>admiral's</td>
</tr>
<tr>
<td>Americanization's</td>
<td>admirals</td>
</tr>
<tr>
<td>Americanizations</td>
<td>admiralty</td>
</tr>
<tr>
<td>...</td>
<td>admiralty's</td>
</tr>
</tbody>
</table>
Spell Checker Example

- Tokenization here: same as previous example FrequencyCounter, just whitespace-delimited strings of non-whitespace. Whitespace = space/tab/newline
  - `new Scanner(System.in)` uses whitespace delimitation, very useful
- Just use this same Scanner setup for a simple spell checker.
- Fill the `Set<String>` dictionary with `M` words
- Loop over `N` tokens, one for each such word, check it using `dictionary.contains(word);` // `O(1)` or `O(log M)`
- Performance: `O(N)` or `O(NlogM)`
TreeMap features

• You don’t have to use the natural (compareTo) order of elements in a TreeMap or TreeSet.
• Instead, you can create a Comparator object to be used in the comparison, and pass it to the TreeMap or TreeSet constructor.
• A comparator takes two objects and returns 0 if they are equal, positive or negative value otherwise.
• Comparators are discussed in S&W pp. 338-339.
• Comparator<T> has method int compare(T o1, T o2)
• They know how to compare two T’s, but have code outside of T’s source file.
Comparators, now easily constructed

- Java 8 provides a new easy way to generate a Comparator for the common case where we want to use a single property for comparing two objects.
- Suppose a Student has a gpa property, i.e. private double gpa; with a getter, getGpa().
- Ex: compare Students by gpa, i.e. student.getGpa() value

```java
Comparator<Student> byGpa = Comparator.comparing(Student::getGpa);
```
- Just a single call to the static Comparator method “comparing” does the trick, specifying which getter should be used by a new Java double-colon syntax for a “method reference”
public static void main(String[] args) {
    Comparator<Student> byGpaComp = // new feature: easy Comparator
        Comparator.comparing(Student::getGpa);
    // use comparator in TreeSet constructor to specify order--
    Set<Student> byGpaSet = new TreeSet<Student>(byGpaComp);
    byGpaSet.add(new Student("Ann", 100, 2.3));
    byGpaSet.add(new Student("Ling", 104, 4.0));
    byGpaSet.add(new Student("Dave", 105, 3.1));
    System.out.println(byGpaSet);
    // Use iterator to access them in GPA order--
    for (Student x: byGpaSet) {
        System.out.println(x);
    }
}
Note: This is covered again in the next class, with more details on Comparators.
How does a TreeSet or TreeMap work?

- Basic idea: binary search trees, (fancier trees like AVL or red-black trees available).
- In the binary search tree the keys are held in the tree nodes and guided the lookup down the tree.
- A tree of N nodes has height about $O(\log N)$, and it takes about $\log N$ decisions to find the right leaf node.
FYI: Set using bit-vectors

- Bit-vectors (also known as bitmaps) are a very simple and very fast implementation for an important kind of sets, that is, sets of moderate-size ints.
- We simply set up an array of bits in effect, bits numbered 0 to n-1, and each bit tells whether that number is in the set or not.
- This is really easy in Java, which supports array of boolean, but only slightly harder in C, where we can use an array of int and for each int, store membership info on 32 set elements, for 32-bit ints.
- IntSet implemented by Array of Boolean. Not a JDK Set, works with primitive arguments.
IntSet implementation

IntSet constructor: take in max size, make an array of Boolean of that size, all false.
s.add(i): bits[i] = true;
s.remove(i); bits[i] = false;
s.contains(i): return bits[i];
void retainAll(IntSet b)
{
    for (int i = 0; i < NBITS; i++)
        bits[i] = bits[i] && b.bits[i];
}
Runtime analysis

• This is O(N), but hopefully Java realizes it can be done 32 bits at a time, by CPUs bitwise AND, so the constant is small. Even smaller if we use longs instead of ints.
• Clearly contains, add, and remove are O(1), while union, intersection, difference, min, and equals are O(N).
• As in the hash table analysis, we can assume the array sizes track actual set sizes in use, by smart resizing.
• Thus we say these are O(n) where n is the set size, but we know they are "fast O(n)" because of the small constant.
Comparison of different Collection methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Hashing</th>
<th>BitMap (IntSet only)</th>
<th>Tree</th>
<th>LinkedList</th>
<th>ArrayList</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construct</td>
<td>$O(1)$</td>
<td>$O(N)$</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Contains, add, remove</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
<td>$O(\log N)$</td>
<td>$O(N)$, except add is $O(1)$</td>
<td>$O(N)$, except add is $O(1)$</td>
</tr>
<tr>
<td>union, intersection, difference</td>
<td>$O(N)$</td>
<td>$O(N)$</td>
<td>$O(N)$</td>
<td>$O(N^2)$</td>
<td>$O(N^2)$</td>
</tr>
<tr>
<td>Next, using Iterator</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Set Min, Max</td>
<td>$O(N)$</td>
<td>$O(N)$</td>
<td>$O(\log N)$</td>
<td>$O(N)$</td>
<td>$O(N)$</td>
</tr>
</tbody>
</table>
Limitations, disadvantages, etc.

- **Hashing:** Assumes “good” hashing function, and rehashing, so the table can start small. Need to code hashCode for user-defined elements. When resizing occurs, the add that caused it takes much longer than the other adds: the O(1) for add is an average.
- **BitMap:** Only for Sets of moderate-sized ints. "Fast" O(n) because 32 elements are processed together, small constant in T(n) formula.
- **Trees:** Requires the elements or domain elements have an ordering. Min/Max for Set is very fast, but that speed applies only to the ordering used in building the tree.
Bringing the BitMap implementation to more cases

- another use of computed mappings to 0..n-1
- Since the bitmap gives us the best-looking union, intersection, and difference performance results, we would like to use it over more cases if possible.
- If we have a one-one computed mapping from an element type to the ints 0 to N-1, and its inverse as well, then we can map from e, an element, to its i, an int, use that as a bit number, and determine membership.
- To return an element, we use the inverse mapping to reconstruct e from i.
- Now we have enough info to do a performance analysis of a Set app.
Performance analysis for movie rental

Recall Map of string to object movie → (qoh, subs)
subs = Set of all movies this one can substitute for, assume O(1)
for all movie in M.keySet():
    M.get(movie) to get (qoh, subs) : O(1)
    Set<String> availSubs = cover-set INTERSECT avail
        if empty, gapcount++
N movies, O(1) in each subs, INTERSECT with avail: needs O(1) contains, so O(1)
isEmpty: O(1), so O(1) per movie, O(N) in all.
So T(N) = O(N), assuming O(1) in cover-sets