CS 720 Spring 2003
Homework 2

Due Date: March 5

1. Let M_i and N_i for $i \geq 1$ be the Kripke structures defined in class when we showed that $A[F(p \wedge Xp)]$ is not equivalent to any CTL formula. Prove by induction on $|f|$ the claim that for all CTL formulas f and $i \geq 1$
   (a) if $|f| \leq i$, then $M_i, a_i \models f$ if and only if $N_i, a_1 \models f$;
   (b) if $|f| < i$, then $M_i, b_i \models f$ if and only if $N_i, b_1 \models f$.

2. Let $\mathcal{M}$ be the Kripke structure of Figure 3.12 on page 170 of Huth and Ryan.
   (a) Using the explicit model checking algorithm of Chapter 4 of Clarke, Grumberg and Peled, show that the CTL formula $\mathbf{AG}(t_1 \rightarrow \mathbf{AF}c_1)$ is not true in any state of $\mathcal{M}$.
   (b) Let $F = \{P_1, P_2\}$ be a set of fairness constraints where P_1 is given by the formula $\neg t_1$ and P_2 is given by the formula $\neg t_2$. Using the explicit model checking algorithm with fairness, show that the formula from part (a) is true in every state with these fairness constraints.

3. Huth and Ryan, Exercises 3.7 (page 176), 2.

4. Huth and Ryan, Exercises 3.8 (page 181), 2.

5. Prove that the two formulas AGg and $\neg EF\neg g$ are equivalent under the fairness semantics.

6. On Homework 1, you were asked to show that $\{AG, AU, AX\}$ is an adequate set of connectives. This is false, even though it appears in Huth and Ryan.
   (a) Download the corrections to Huth and Ryan from the homepage for the text.
   (b) Download the paper Huth and Ryan refer to as showing that this claim is wrong.
   (c) The inadequacy of $\{AG, AU, AX\}$ is nver explicitly stated in the paper. Which result in the paper shows that $\{AG, AU, AX\}$ is not adequate?
   (d) How does the inadequacy of $\{AG, AU, AX\}$ follow from this result?