**CS 720 Spring 2003**
**Homework 4**

**Due Date: April 30**

1. Huth and Ryan, Exercises 6.7 (page 341), Problem 8 a,b.
2. Huth and Ryan, Exercises 6.7 (page 341), Problem 11.
3. Huth and Ryan, Exercises 6.8 (page 343), Problem 1 a,b.
4. Huth and Ryan, Exercises 6.9 (page 346), Problem 1 a.
5. Huth and Ryan, Exercises 6.10 (page 349), Problem 1.
6. Huth and Ryan, Exercises 6.11 (page 354), Problem 1.
7. Huth and Ryan, Exercises 6.12 (page 357), Problem 2.
8. Prove the following two identities directly.

$$\begin{aligned} \mathbf{AG}f_1 &= \nu Z.f_1 \wedge \mathbf{AX}Z \\ \mathbf{AF}f_1 &= \mu Z.f_1 \vee \mathbf{AX}Z \end{aligned}$$

9. Let $\mathcal{M}$ be the Kripke structure of Figure 3.12 on page 170 of Huth and Ryan. Using the LTL model checking algorithm of Section 6.7 of Clarke, Grumberg and Peled, show that the LTL formula $\mathbf{AG}(\neg(c_1 \wedge c_2))$ is true in every state of the Kripke structure.
10. Consider the tableau on page 89 of Clark, Grumberg and Peled. Let $\pi'$ be the path $(3, 3, 3, 3, \ldots)$ in $T$. Then $T, \pi' \models \neg g$, where $g$ is the formula $(\neg heat)\mathbf{U}close$. According to Theorem 4, there must be a path $\pi$ in $T$ that starts in a state in $sat(\neg g)$ such that $label(\pi')|_{AP_{\neg g}} = label(\pi)$. This path is not $\pi'$ itself since $3 \notin sat(\neg g)$. What is the path $\pi$ and how do you find it using the method of the proof of Theorem 4?
11. Let $\tau$ be a monotonic predicate transformer on a set $S$. Show that

$$\mu z.\tau(Z) = \overline{\nu z.\overline{\tau(\overline{Z})}}.$$

In other words, show that if $\tau'(Z) = \overline{\tau(\overline{Z})}$, then $\tau'$ is also montonic and the complement of the greatest fixpoint of $\tau'$ is the least fixpoint of $\tau$.