allez

Chapter 4

Description of Noticed Orbits in Simple Real Lie Algebras

Introduction

Let $\mathfrak{g}$ be a simple real Lie algebra, and E a nilpotent element of $\mathfrak{g}$. Since we are dealing with conjugacy classes, we may choose E in a KS-triple (H, E, F). Thus, the Kostant-Sekiguchi correspondence gives a bijection between the G-nilpotent orbits in $\mathfrak{g}$ and the $K_{\mathbb{C}}$-nilpotent orbits in $\mathfrak{p}_{\mathbb{C}}$. Recall that e is noticed if and only if the reductive subalgebra $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} = 0$, where (x, e, f) is a normal triple of $\mathfrak{g}_{\mathbb{C}}$. Our goal is to give a simple procedure to identify noticed nilpotent orbits of $\mathfrak{g}$. That is the ones that correspond to noticed nilpotent orbits in $\mathfrak{p}_{\mathbb{C}}$.

4.1 Simple exceptional real Lie algebras

D. Djoković **[D1, D2]** (see also **[C-Mc]**) has computed the reductive centralizer for all real nilpotent orbits in the case where $\mathfrak{g}$ is an exceptional simple real Lie algebra. The results are given in several tables, one for each algebra. Hence the noticed orbits can be easily identified from Djoković's tables. They are the ones for which $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} = 0$. The last two colums give the isomorphism type of $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$ and $\mathfrak{g}^{(H,E,F)}$ respectively. A study of the tables reveals that :

$$E_{6(2)},\ E_{6(6)},\ E_{7(7)},\ E_{8(8)},\ F_{4(4)},\ G_{2(2)}$$

are the only exceptional simple real Lie algebras to admit noticed nilpotent orbits. They are *quasi-split.*

A real form, $\mathfrak{g}$, of $\mathfrak{g}_{\mathbb{C}}$ is called *quasi-split* if there is a subalgebra, $\mathfrak{b}$, of $\mathfrak{g}$ such that $\mathfrak{b}_{\mathbb{C}} = \mathfrak{b} + i\mathfrak{b}$ is a Borel subalgebra of $\mathfrak{g}_{\mathbb{C}}$.

The following proposition characterizes quasi-split real forms.

Proposition 4.1.0 (Rothschild). *$\mathfrak{g}$ contains a regular nilpotent iff $\mathfrak{g}$ is quasi-split iff $\mathfrak{g}$ contains a regular semisimple H such that ad_H has all real eigenvalues.*

Proof. See L. Preiss Rothschild. **[Rot]**

An element z of $\mathfrak{g}$ is said to be regular if dim $G{\cdot}z \geq$ dim $G{\cdot}y$ for all y in g.

Hence,

Theorem 4.1.1. *An exceptional simple real Lie algebra is quasi-split if and only*

if it contains a noticed nilpotent element.

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This is not the case in general, as we shall see below. Now we turn our attention to the classical algebras.

4.1 Simple classical real Lie algebras

Let $\mathfrak{g}$ be a real classical Lie algebra. It is known **[S-S, B-Cu, C-Mc]** that the nilpotent orbits of G in $\mathfrak{g}$ are parametrized by signed Young diagrams. Let (H, E, F) be a standard triple in $\mathfrak{g}$. Denote by V the standard repesentation of $\mathfrak{g}$, regarded as a $\mathfrak{sl}_2$-module under the action of the real Lie algebra generated by (H, E, F). Then

$$V = \bigoplus_{r \geq 0} M(r)$$

where each $M(r)$ is a direct sum of irreducible $(r+1)$-dimensional $\mathfrak{sl}_2$-modules. Let $[d_1, d_2, \ldots, d_k]$ be a partition of $\dim_{\mathbb{R}} V$. Then the d_i's are exactly the dimensions of the irreducible summands of V (see **[C-Mc]**). For $r \geq 0$, denote by $H(r)$ the highest weight space in $M(r)$. From the $\mathfrak{sl}_2$ theory we have

$$\dim H(r) = \text{mult}(\rho_r, M(r))$$

where ρ_r is the irreducible representation of $\mathfrak{sl}_2$ of highest weight r. If $\mathfrak{g}$ is not $\mathfrak{su}^*_{2n}$ or $\mathfrak{sl}_n(\mathbb{R})$, then V carries a $\mathfrak{g}$-invariant form $\langle .,. \rangle$ which induces a non degenerate bilinear form $(.,.)_r$ on $H(r)$ as follows:

$$\text{if } u, v \in H(r) \text{ then } (u, v)_r = \langle u, F^r v \rangle$$

It turns out that the induced forms $(.,.)_r$ determine the ambient form $\langle .,. \rangle$ uniquely. **[C, Mc]**

For example if we assume that $\langle .,. \rangle$ is symmetric, it is enough to illustrate that claim on one irreducible module $L(r)$ of $M(r)$. Choose a basis:

$$e_1, e_2, \ldots, e_r, e_{r+1} \text{ of } L(r).$$

such that e_i spans the $r - 2(i-1)$ weight space of H in $L(r)$.

Then clearly

$$(e_1, e_1)_r = \langle e_1, F^r e_1 \rangle = -\langle Fe_1, F^{r-1} e_1 \rangle = \langle F^2 e_1, F^{r-2} e_1 \rangle = (-1)^l \langle F^l e_1, F^{r-l} e_1 \rangle.$$

The induced form $(.,.)_r$ will play a key role in our parametrization of the noticed nilpotent orbits of the classical real Lie algebras.

The reductive centralizer $C = \mathfrak{g}_{\mathbb{C}}^{(x,e,f)}$ can be described as a direct sum of simple complex Lie algebras **[S-S, Ca, C-Mc]**. The nilpotent orbits of $G_{\mathbb{C}}$ in $\mathfrak{g}_{\mathbb{C}}$ are also parametrized by certain partitions. Let r_i be the number of parts equal to i in the description of the partition associated to e.

For type A_n

$$C = (\bigoplus_i A_{r_i - 1}) \oplus T_k$$

where $k = (\text{No. of } r_i) - 1$, and T_k is a torus of dimension k.

For type C_n

$$C = (\bigoplus_{i \text{ odd}} C_{r_i/2}) \oplus (\bigoplus_{i \text{ even},\ r_i \text{ even}} D_{r_i/2}) \oplus (\bigoplus_{i \text{ even},\ r_i \text{ odd}} B_{(r_i-1)/2}) .$$

For type B_n and D_n

$$C = (\bigoplus_{i \text{ even}} C_{r_i/2}) \oplus (\bigoplus_{i \text{ odd},\ r_i \text{ even}} D_{r_i/2}) \oplus (\bigoplus_{i \text{ odd},\ r_i \text{ odd}} B_{(r_i-1)/2}) \ .$$

In the above formulae D_1 must be interpreted as a 1-dimensional torus T_1 wherever it occurs.

To classify the noticed nilpotent $K_{\mathbb{C}}$-orbits in $\mathfrak{p}_{\mathbb{C}}$ we shall proceed as follows: maintaining the above notations, for each real algebra we will determine which signed Young diagrams will force $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$ to be zero. Since

$$\mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \oplus \mathfrak{p}_{\mathbb{C}}^{(x,e,f)},$$

we should be able to carry out our analysis on each real algebra separately by indentifying which signed Young diagrams that produce a real centraliser $\mathfrak{g}^{(H,E,F)}$ in $\mathfrak{p}$. We are using Helgason's [**He**] realizations of the classical real algebras.

Type A_n

1. $\mathfrak{sl}_n(\mathbb{R})$**.**

Let $\mathfrak{g} = \mathfrak{sl}_n(\mathbb{R})$. Then $\mathfrak{k} = \mathfrak{so}_n$ and $\mathfrak{p}$ = set of $n \times n$ symmetric real matrices. The nilpotent orbits of $\mathfrak{g}$ are parametrized by signed Young diagrams of size n, except that partitions having only even terms correspond to two orbits, denoted as usual by I and II. [**C-Mc**]. Here $\mathfrak{g}_{\mathbb{C}} = \mathfrak{sl}_n(\mathbb{C})$ and its nilpotent orbits are also parametrized by partitions of n [**C-Mc**]. Moreover the Young diagram of the complexification of a real orbit of $\mathfrak{g}$ is obtained by omitting the signs and the numeral in the case of "even" orbits, that is, orbits parametrized by partitions with even parts only. Therefore for an orbit parametrized by a given partition $\bar{d}$ to be noticed, that is for $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$ to be trivial, all the parts of $\bar{d}$ must be distinct since otherwise C will have a summand of type A_m an any real form of A_m has a compact part. Hence if e is

noticed then the associated partition has distinct parts and

$$\mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = T_k$$

We, now show how to construct the torus part T_k of the centralizer C in $\mathfrak{p}_{\mathbb{C}}$.

Let $\bar{d} = [d_1, d_2, \ldots, d_{k+1}]$ be the partition of n with distinct parts associated with e. Consider the set of $n \times n$ diagonal matrices D_i for $1 \leq i \leq k$ defined as follows :

Let $c_i = (\sum_{j=1}^{i} d_j) - d_i$. Then D_i is the $n \times n$ diagonal matrice with d_i's consecutive 1's starting at row $c_i + 1$ and the last d_{k+1} entries are all equal to $\frac{-d_i}{d_{k+1}}$. For example

$$D_1 = \begin{pmatrix} I_{d_1} & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & 0 & \ldots & \ldots & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \frac{-d_1}{d_{k+1}} & \vdots & 0 \\ & & & & \ddots & \\ 0 & 0 & \ldots & \ldots & & 0 \\ 0 & 0 & \ldots & \ldots & \ldots & \frac{-d_1}{d_{k+1}} \end{pmatrix}$$

Here I_{d_i} denotes the $d_i \times d_i$ identity matrix. The diagonal matrices D_i have trace zero and commute with (x, e, f). Furthermore , by construction they are independent and generate T_k. Hence

$$\dim \mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = k.$$

But

$$\dim \mathfrak{p}_{\mathbb{C}}^{(x,e,f)} \geq k.$$

Thus by dimensionality considerations, we obtain:

$$\mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = \mathfrak{p}_{\mathbb{C}}^{(x,e,f)}.$$

2. $\mathfrak{su}(p,q)$**.**

Let $\mathfrak{g} = \mathfrak{su}(p,q)$. Then (see Helgason, [**He**])

$$\mathfrak{k} = \left\{ \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} | A \in \mathfrak{u}(p), B \in \mathfrak{u}(q), trace(A+B) = 0 \right\}.$$

The nilpotent orbits of $\mathfrak{su}(p,q)$ are parametrized by signed Young diagrams of signature (p,q) [**C-Mc**]. The complexified complex orbits of these real orbits are parametrized by partitions of $n = p+q$. Let $\bar{d} = [d_1, d_2, \ldots, d_{k+1}]$ be a partition of n with distinct parts. Assume that $\bar{d}$ represents a nilpotent orbit of $\mathfrak{g}$. Then it is clear that the diagonal matrices of the form $\sqrt{-1}D_i$ where D_i is the matrix defined above generate T_k. Again in this case we have:

$$\dim\ \mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = k.$$

But

$$\dim\ \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \geq k.$$

Thus by dimensionality, we obtain:

$$\mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = \mathfrak{k}_{\mathbb{C}}^{(x,e,f)}.$$

3. $\mathfrak{su}^*_{2n}$**.**

Let $\mathfrak{g} = \mathfrak{su}^*_{2n}$. The nilpotent orbits of $\mathfrak{g}$ are parametrized by partitions of n. To obtain the partitions associated with their complexification we have to replace each row of their Young diagrams with two copies of itself [**C-Mc**]. Hence the complexified orbits are parametrized by partitions of $2n$ with no distinct parts.

Note that the partition $\bar{d}$ has only one part if and only if the reductive centralizer is zero. In which case the orbit is distinguished in Bala-Carter's sense. Hence it is noticed.

Since any partition with repeated parts will produce a centralizer with summands of type A_l, we have the following theorem:

Theorem 4.2.1. *The non-zero noticed nilpotent orbits of* $\mathfrak{sl}_n(\mathbb{R})$ *are parametrized by partitions of* n *with distinct parts. If such a partition is even then it corresponds to two orbits labeled by I and II. The algebra* $\mathfrak{su}^*_{2n}$ *has no non-zero noticed nilpotent orbit. Also* $\mathfrak{su}(p, p+1)$ *has exactly one non-zero noticed nilpotent orbit, which is parametrized by a one-row signed Young diagram of signature* $(p, p+1)$*,* $\mathfrak{su}(p,p)$ *has exactly two non zero noticed nilpotent orbit, each is parametrized by a one-row signed Young diagram of signature* (p,p) *. If* $|p-q| \geq 2$ *then* $\mathfrak{su}(p,q)$ *has no non-zero noticed nilpotent orbits.*

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Types B_n and D_n

1. $\mathfrak{so}(p,q)$**.**

Let $\mathfrak{g} = \mathfrak{so}(p,q)$. Then the nilpotent orbits in $\mathfrak{g}$ are parametrized by signed Young diagrams of signature (p,q) such that rows of even length occur with even multiplicity and have their left most boxes labeled $+$. Some of these diagrams get Roman numerals attached to them as follows. If all rows have even length, then two Roman numerals, each I or II are attached. If at least one row has odd length and all such rows have an even number of boxes labeled $+$, or all such rows have an even number of boxes labeled -, then one numeral I or II is attached. **[C-Mc]**. Moreover $\mathfrak{g}_{\mathbb{C}} = \mathfrak{so}_{p+q}(\mathbb{C})$ and its nilpotent orbits are also parametrized by partitions of $p+q$ in which even parts appear with even multiplicity **[C-Mc]**. To obtain the Young diagram of the complexification of a real orbit of $\mathfrak{g}$ we omit the signs. If the associated partition is very even that is every part is even and appears with even multiplicity we omit the first numeral. If it is not very even we omit the numeral. Therefore for an orbit parametrized by a given partition $\bar{d}$ to be noticed, that is for $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$ to be trivial, all the parts of $\bar{d}$ must be odd and repeated at most twice. Otherwise C will have a summand of type B_l or C_m or D_n with $n \leq 2$. Hence if e is noticed then

$$\mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = T_k.$$

Note that k is the number of parts with multiplicity 2. Cleary if $k = 0$ then e is distinguished.

We, now show how to construct the torus part T_k of the centralizer C in $\mathfrak{p}_{\mathbb{C}}$. This will make it possible to identify which of the nilpotents whose partitions have all parts repeated at most twice are noticed.

Let $\bar{d} = [d_1, d_2, \ldots, d_l]$ be a partition of $p+q$ of signature (p,q) which consists of odd parts repeated at most twice. Let (H,E,F) be a KS-triple associated with $\bar{d}$. Then the standard representation V of $\mathfrak{g}$ can be decomposed as a direct sum of

irreducible $\langle H, E, F\rangle$-modules of weight $r_i = d_i - 1$. The centralizer of $\mathfrak{g}$ preserves each $M(r_i)$. It also preserves the $\mathfrak{g}$-invariant symmetric form, $\langle .,. \rangle$, carried by V. Since all the r_i's are even, (see [**C-Mc**]), $(.,.)_{r_i}$ is symmetric and the signature of $\langle .,. \rangle$ on $M(r_i)$ is obtained by starting with the signature of $(.,.)_{r_i}$ and replacing each $\pm$ sign s by an alternating sequence of signs of length d_i, beginning with s. Therefore if d_i has multiplicity 1 there are only two possible labellings since the one dimensional highest weight space is labelled either with a + or a -. However if d_i is repeated twice then we have three possible ways of labelling $M(r_i)$. It is useful to consider the following example:

Assume that we are trying to label $M(2)$ from the standard representation of $\mathfrak{so}(3,3)$ with regard to the partition $[3,3]$. Since symmetric real forms are equivalent up to signature, we can rearrange the basis of V such that :

$$M(2) = \langle e_1, e_2, e_3 \rangle \oplus \langle e_4, e_5, e_6 \rangle$$

it is understood that each basis vector denotes a one dimensional weight space in an $\mathfrak{sl}_2$ irreducible module. In our notation (e_1, e_4) form a basis of $H(2)$. Clearly we can label M(2) in 3 different ways:

$$\begin{array}{ccc} + - + & + - + & - + - \\ - + - & + - + & - + - \end{array}$$

The first row represents the labels of e_1, e_2 and e_3. The second row represents the labels of e_4, e_5 and e_6

In general if the part d_l is represented twice in the partition $\bar{d}$, then $M(r_l)$ is the direct sum of two irreducible $\mathfrak{sl}_2$-modules of highest weight r_l. Furthermore V has

a basis where:

$$M(r_{_l}) = \langle e_k, e_{k+1}, \ldots, e_{k+r_l} \rangle \oplus \langle e_{k+r_l+1}, e_{k+r_l+2}, \ldots, e_{k+2r_l+1} \rangle$$

Once again (e_k, e_{k+r_l+1}) is a basis for $H(r_{_l})$ and $(e_{k+r_l}, e_{k+2r_l+1})$ is a basis for $H(-r_{_l})$.

Assume that $M(r_l)$ is labelled as follows:

$$+ - + - + \ldots \; - +$$

$$- + - + - \ldots \; + -$$

Let Z be a $(p+q) \times (p+q)$ matrix such that :

$$Ze_i = e_{i+r_l+1} \qquad \text{and} \qquad Ze_{i+r_l+1} = e_i \qquad \text{if} \qquad k \leq i \leq k + r_l \qquad else \qquad Ze_i = 0.$$

Then

$$(e_k, Ze_k)_{r_l} = (e_k, e_{k+r_l+1})_{r_l} = -(Ze_k, e_k)_{r_l} = 0,$$

$$(e_{k+r_l+1}, Ze_{k+r_l+1})_{r_l} = (e_{k+r_l+1}, e_k)_{r_l} = -(Ze_{k+r_l+1}, e_{k+r_l+1})_{r_l} = 0,$$

$$(e_k, Ze_{k+r_l+1})_{r_l} = (e_k, e_k)_{r_l} = -(e_{k+r_l+1}, e_{k+r_l+1})_{r_l} = -(Ze_k, e_{k+r_l+1})_{r_l}.$$

Hence $(\,,\,)_{r_l}$ is invariant under Z and so is the ambient form $\langle .,. \rangle$. Furthermore it easy to see that Z commutes with (H, E, F). Therefore Z generates a one dimensional factor of the centralizer $\mathfrak{g}^{(H,E,F)}$. By definition Z is a Hermitian matrix. So all its eigenvalues are real and it lies in the vector part of a Cartan subalgebra of $\mathfrak{g}$. We conclude that Z is G-conjugate to some element of $\mathfrak{p}$. (See Helgason [**He**])

A similar analysis can be carried out for the other two labellings. In both cases one defines Z as follows:

$$Ze_i = e_{i+r_l+1} \qquad \text{and} \qquad Ze_{i+r_l+1} = -e_i \qquad \text{if} \qquad k \leq i \leq k + r_l \qquad else \qquad Ze_i = 0.$$

It is easy to verify that Z preserves $(,)_{r_l}$. Since Z is a skew-hermitian matrix, all its eigenvalues are imaginary. Therefore Z is G conjugate to some element of $\mathfrak{k}$. (See Helgason [**He**])

Observe that the matrices Z described above commute and are independent. Also there are exactly k such matrices for each orbit. Therefore they generate T_k. Hence

$$\dim \mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = k.$$

In the first case all the Z's are in $\mathfrak{p}_{\mathbb{C}}$. Hence

$$\dim \mathfrak{p}_{\mathbb{C}}^{(x,e,f)} \geq k.$$

Thus by dimensionality, we obtain:

$$\mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = \mathfrak{p}_{\mathbb{C}}^{(x,e,f)}.$$

In the two other cases all the Z's are in $\mathfrak{k}_{\mathbb{C}}$. Hence

$$\mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = \mathfrak{k}_{\mathbb{C}}^{(x,e,f)}.$$

2. $\mathfrak{so}_{2n}^*$**.**

Let $\mathfrak{g} = \mathfrak{so}_{2n}^*$. Then the nilpotent orbits are parametrized by signed Young diagrams

of size n and any signature in which rows of odd length have their leftmost boxes labeled + **[C-Mc]**. Let $\bar{d} = [d_1, d_2, \ldots, d_l]$ be a partition of n which consists of distinct odd parts only. Now $\mathfrak{so}^*_{(2n)}$ acts on a $2n$ dimensional complex space V and can be defined as the subalgebra of $\mathfrak{so}(2n, \mathbb{C})$ that leaves invariant a skew-hermitian form $\langle ., . \rangle$ on V. With the above notation V can be seen as a direct sum of M_{r_i}'s and each of them is made of two irreducible modules of dimension d_i. Since all the r_i's are even, (see [**C-Mc**]), $(.,.)_{r_i}$ is skew-hermitian. Define $Z.e_k = \sqrt{-1}e_k$ for $e_k \in M_{r_i}$ and zero elsewhere. Then clearly Z preserves H_{r_i} and belongs to $g^{(E,H,F)}$. But Z is skew-hermitian matrix and consequently is G-conjugate to some element of $\mathfrak{k}$. Consequently

$$\dim \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \geq k.$$

Thus by dimensionality, we obtain:

$$\mathfrak{g}_{\mathbb{C}}^{(x,e,f)} = \mathfrak{k}_{\mathbb{C}}^{(x,e,f)}.$$

From the definition of $\mathfrak{g}_{\mathbb{C}}^{(x,ef)}$, the description of the nilpotent orbits of $\mathfrak{so}(p,q)$ and $\mathfrak{so}^*_{2n}$ in **[C-Mc]** supplemented by the the above discussion we have the following theorem:

Theorem 4.2.2. *The non-zero noticed nilpotent orbits of* $\mathfrak{so}(p,q)$ *are parametrized by Young diagrams of signature (p,q) such that :*

(1) all rows are odd and can be repeated at most twice,

(2) two rows of the same length must have their leftmost boxes labeled by different signs,

(3) if all the rows have an even number of boxes labeled +, or all the rows have

an even number of boxes labeled −, then one numeral I or II is attached.

There are no non-zero noticed nilpotent orbits in $\mathfrak{so}^*_{2n}$.

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4.2.3 Types C_n.

Let $\mathfrak{g} = \mathfrak{sp}(p,q)$. Then the nilpotent orbits in $\mathfrak{g}$ are parametrized by signed Young diagrams of signature (p,q) such that rows of even length have their left most boxes labeled +.**[C-Mc]**

Nilpotent orbits of $\mathfrak{sp}_n(\mathbb{R})$ are parametrized by signed Young diagrams of size $2n$ and any signature in which odd rows appear with even multiplicity and begin with +. **[C-Mc]**

Also $\mathfrak{g}_{\mathbb{C}} = \mathfrak{sp}_n(\mathbb{C})$ and its nilpotent orbits are parametrized by partition of $2n$ in which odd parts occur with even multiplicity.
A discussion similar to the one given in the previous section gives the next theorem.

Theorem 4.2.3. *The non-zero noticed nilpotent orbits of* $\mathfrak{sp}_n(\mathbb{R})$ *are parametrized by signed Young diagrams of size* $2n$ *such that:*

(1) all rows are even an can be repeated atmost twice,

(2) two rows of the same length must have their leftmost boxes labeled with different signs.

There are no non-zero noticed nilpotent orbits in $\mathfrak{sp}(p,q)$.

■

We have seen that the only exceptional simple real Lie algebras admitting nilpotent orbits are the quasi-split ones. This is not true in the case of the classical simple real Lie algebras, as the following example indicates.

Let $\mathfrak{g} = \mathfrak{so}(6,3)$. Then from theorem 4.2.2, the orbit parametrized by the signed Young diagram:

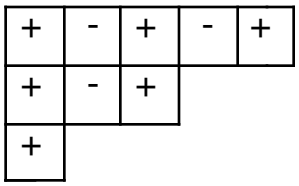

figure 1

is noticed but $\mathfrak{g}$ is not quasi-split.

Remark: We note that the noticed orbits of the simple real Lie algebras of type B_n, C_n and D_n are even. This can be seen through an analysis of the weighted Dynkin diagram associated with their complex counterparts (See **[C-Mc]**). Therefore our classification, when restricted to the above algebras, is similar to the Bala-Carter classification.