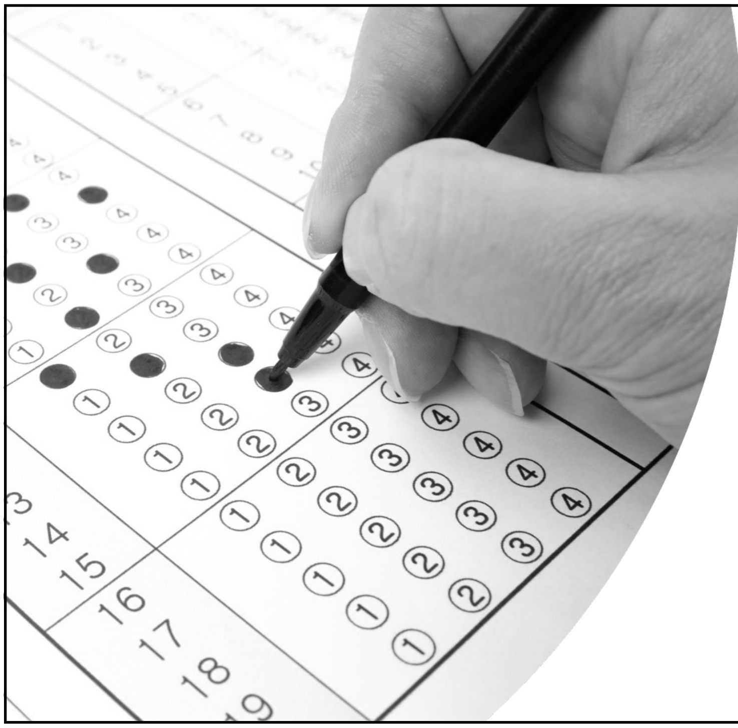

1

Course Evaluation

Get the unique link on Piazza

1

Exam structure

Question 1: (15 points)
- True/False or Yes/No

Question 2: (15 points)
- Probability problems

Question 3: (15 points)
- Tree Quizzes

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- Recurrence Relation

Question 5: (10 points)
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Question 7: (10 points)
- Easy & Simple computation

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- Conditional Probability

2

Final Exam policy

- ❑ **Camera on!** Webcams will be used for proctoring
- ❑ Showing Student ID at the start of the exam.
- ❑ Please login 15 minutes early (9:45am)
- ❑ Private message in the chat box
- ❑ **Exactly 10 minutes** to submit the solution (GradeScope)
- ❑ No internet resources; No collaborations; Opened book
- ❑ Submissions
  - Handwriting → JPG/JPEG images
  - MS Word/Latex editor → PDF file
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3

Applied Discrete Mathematics (CS220)

Review and practice

4

Contents

Propositional Logic

- Propositions and Logical operator

$$\neg, \wedge, \vee, \oplus, \rightarrow, \leftrightarrow$$

- Propositional Formula and Its classification
  - Contingence
  - Tautology
  - Contradiction

Propositional Logic

- How to determine whether a compound proposition is a tautology/contradiction/contingence?
  - Using truth table
  - Using logical equivalence rules
- Propositional equivalent?
  - Some important equivalences
- Valid of Argument
  - Some important equivalences
  - Rules of Inference

Propositional Logic

- Predicates and Quantifiers
  - Universal quantifier ∀
  - Existential quantifier ∃
- Logical Equivalence
  - De Morgan's laws for predicates
  - Quantifiers

$$\forall x\big(P(x) \land Q(x)\big) \equiv \forall x P(x) \land \forall x Q(x)$$
$$\exists x\big(P(x) \lor Q(x)\big) \equiv \exists x P(x) \lor \exists x Q(x)$$

Boolean Algegra

- Boolean operators
  - Boolean complement
  - Boolean sum
  - Boolean product
- Boolean functions and expressions
  - Minterm method to determine Boolean expression
  - Karnaugh Map (K-Map) metho to find equivalent minimal boolean expression
  - Circuit gates

9

Set Theory

- Set and Set operators
  - Set operators: ∩,∪, –
- Subset(⊆), proper subsets(⊂)
- Power set (2^S)
- Set partitions
- Cardinality of set
  - Principle of inclusion-Exclusion

10

Relations

- Relations and their representing
  - $R \subseteq A \times B$
  - Representing methods: Set, Matrix, Graph
- Properties of relations
  - Reflexive, Symmetric, Transitive, Asymmetric
- Combining relation
  - Composite
  - Inverse

Relations

- Closures of relations
  - Reflexive closure
  - Symmetric closure
  - Transitive closure
- Equivalence relation
  - Equivalence relation: reflexive, symmetric, and transitive
- Partial ordering and Poset
  - Reflexive, Transitive, Antisymmetric
  - Hasse Diagram

Functions

- ❑ Terminologies
  - Domain, Codomain, Image, Preimage, Range
- ❑ Properties of functions
  - Injective (one-to-one)
  - Surjective (onto)
  - Bijective
- ❑ Composition and Inversion

Recurrence Relations

- ❑ Recurrence relations
- ❑ Solving recurrence relations
  - First form: $a_n = k \cdot a_{n-1}$; $a_0 = C$

    $a_n = C \cdot k^n$
  - Second form: $a_n = a_{n-1} + k$; $a_0 = C$

$$a_n = C + \sum_{i=1}^{n} k$$

Linear homogenous recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$$

1. Determine characteristic equation
2. Find the roots of characteristic equation
   - Case 1: Those roots are different $r_1, r_2, \ldots, r_k$.
   $$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n + \cdots + \alpha_k r_k^n$$
   - Case 2: Identical roots $\overbrace{r_1 r_1 \ldots r_1}^{t\ roots}\overbrace{r_2, r_2 \ldots r_2}^{s\ roots}$, where $t + s = k$
   $$a_n = \alpha_1 r_1^n + \alpha_2 n r_1^n + \cdots + \alpha_t n^{t-1} r_1^n + \beta_1 r_2^n + \beta_2 n r_2^n + \cdots + \beta_s n^{s-1} r_2^n$$
3. Find α_i based on initial values

Algorithm and Complexity

- Pseudo code
- Complexity of Algorithm and Big-O
  - Common Big-O
- Rules for Big-O

Induction and Recursive algorithm

- ❑ Mathematical induction
- ❑ Strong induction (second principle of mathematical induction)
- ❑ Recursive algorithms

Integer property

- ❑ Divisibility theorems
- ❑ Primes and Prime factorization
  - Find *gcd, lcm* using prime factorization
  - Relatively prime integers
- ❑ Modular arithmetic and Congruence
  - Euclidean algorithm to find *gcd*
- ❑ Integer representations
  - Binary expansion, Hexadecimal expansion, Octal expansion

Counting

- Basic principle
  - The sum rule, the product rule
  - The subtraction rule (inclusion-exclusion), the division rule
  - Using tree diagram
- Pigeonhole Principle
- Permutations and Combinations

Discrete Probability

- Definition probability
- Complement events
- Inclusion-Exclusion in Discrete probability
- Conditional probability
- Independence events
- Random variables and Expected values

Graph

- Types of graphs
- Terminologies
  - Sub-graph
  - Degree of vertices
  - Isomorphic graphs
  - Path, circuit/cycle
- Special graphs
  - K_n, $K_{n,m}$, C_n, W_n, Q_n

Graph

- Euler path & Euler circuit
  - Algorithm finding Euler circuit
- Hamilton path & Hamilton circuit
- Shortest path
  - Dijkstra algorithm

Tree

- ❑ Terminologies
  - Tree, Forest
  - Leaf, internal vertices
  - Height of tree
- ❑ Tree traversal
- ❑ Minimal spanning tree
  - Prim's algorithm
  - Kruskal algorithm

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Practice

True or false (why?)

a) $f(x) = 2x + 1, g(x) = x^2 + 4$ then $f \circ g(2) = 29$
b) $\forall x\big(P(x) \vee Q(x)\big) \equiv (\forall x P(x)) \vee (\forall x Q(x))$
c) If $a \equiv 11 \ (mod\ 19), b \equiv 3 \ (mod\ 19), c \equiv 7a + 3b(mod\ 19)$ then c = 10
d) If G is a simple graph with 50 vertices, the maximum edges 1225
e) If T is a binary tree with 41 vertices, its minimum height is 5
f) If T is a full binary tree with 111 vertices, its maximum height is 50.
g) Every full binary tree with 51 vertices has 26 leaves.
h) Every full binary tree with 60 leaves has 120 vertices.
i) Every full binary tree with 75 vertices has 37 internal vertices
j) A full 3-ary tree with 100 internal vertices has 300 vertices.

Practice

Cardinality

How many distinct elements does the set S contain in each case?

a) $S = \{7, 2, 3\} \cup \{3, 1, 2\}$

b) $S = \{(x, y), (y, z), (z, z)\} \cap \{(y, x), (z, z), (y, y)\}$

c) $S = \{A \mid (A \subseteq \{1, 2, 3, 4\}) \wedge (|A| = 5)\}$

d) $S = \{x \mid x2 + 2x = 8; x \text{ is a real number}\}$

e) $S = \{(a, b) \mid a < b; a, b \in \{1, 2, 3\}\}$

f) $S = E$, where $G = (V, E)$ is a tree and $|V| = 5$

g) $S = \{G \mid G \text{ is a simple graph with 4 vertices}\}$

h) $S = \{R \mid R \text{ is a reflexive relation on } \{0, 1\}\}$

i) $S = \{n \mid (n \text{ is prime}) \wedge (n \bmod 2 = 0)\}$

j) $S = \{a, b, c, e\} - \{b, c, d\}$

Practice

Recurrence relations practice

Somewhere in the forests, scientists discovered two rare species of animals named **V** and the **S**. On their first encounter with these animals, the scientists found five animals of each species. One year later, the scientists returned and then found five **V** and 13 **S**. The scientists somehow devised formulas for the populations vn and sn, denoting the number of **V** and **S**, respectively, in year n, for $n \geq 2$: $vn = n \cdot vn\text{-}1 sn = 4sn\text{-}1 + 5sn\text{-}2$

a) Let us define that the species were discovered in year 0, and the second counting was done in year 1. Use the above formulas to predict the populations vn and sn in the years n = 2, 3, 4, and 5.

b) Find explicit formulas for vn and sn, $n \geq 2$, that do not require iteration. Check the correctness of your formulas using some of the results obtained in a).

c) Describe the growth of vn and sn using the **big-O** notation for each of them. In each estimate O(f(n)), f(n) should be the most suitable function chosen from the following ones: log n, n, n log n, $n2$, $n3$, $2n$, $3n$, $4n$, $5n$, $6n$, $n!$, nn.

d) In the year 2050, will there be more V than S, given that the populations develop as predicted? Or will there be more S than V? ***Do not try to compute the actual numbers!*** *Just tell which species you think will have the larger population, and give the reason why you think so.*

Practice

Probability Practices

a) There is an urn containing four blue balls and four red balls. We randomly draw four balls from this urn without returning any balls. What is the probability that all of the four balls that we drew are blue?

b) There are two urns, each of them containing two blue balls and two red balls. We randomly draw two balls from the first urn and then randomly draw two balls from the second urn, without returning any balls. What is the probability that all of the four balls that we drew are blue?

c) There are four urns, each of them containing one blue ball and one red ball. We randomly draw one ball from each urn without returning any balls. What is the probability that all of the four balls that we drew are blue?