Attendance Code →

8/11/2022 : Probability Practicing
(Section 1)

1) Event space : All the posibility can happen. $|S|$
2) Counting each. Number of Event E happen $|E|$

$$P(E) = \frac{|E|}{|S|} \qquad 0 \le P(E) \le 1$$

3) If there are some different events $e_1, e_2 \cdots e_n$ can happen in S

$$p(e_1) + p(e_2) + \cdots + p(e_n) = 1$$

4) $E_1, E_2 \Rightarrow$ $p(E_1 \cup E_2) = P(E_1) + P(E_2) - p(E_1 \cap E_2)$

1. **What is the probability that a card selected at random from a standard deck of 52 cards is an ace?**

$|S| = 52$

We have 4 Aces $|E| = 4$

$$P(E) = \frac{|E|}{|S|} = \frac{4}{52}$$

$$P(E) = \frac{1}{13} \simeq 7.69\%$$

2. **What is the probability that a fair die comes up six when it is rolled?**

$|S| = 6$

$|E| = 1$ $\Rightarrow$ $P(E) = \frac{1}{6} \simeq 0.166\ldots$

3. **What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?**

$|S| = 100$

$|E| = 50$

$$P(E) = \frac{50}{100} = \frac{1}{2} = 0.5$$

4. What is the probability that a randomly selected day of a leap year (with 366 possible days) is in April?

$|S| = 366$

$|E| = 30$

$P(E) = \frac{30}{366} = 5/61 \simeq 0.082.$

5. What is the probability that the sum of the numbers on two dice is even when they are rolled?

$D_1\ D_2 = \{(1,1), (1,2), \ldots, (1,6), (2,1), \ldots (2,6)\}$

$|S| = 6 \times 6 = 36$

$|E| = 18$ both of two values are even / odd

$\Rightarrow P(E) = \frac{|E|}{|S|} = \frac{18}{36} = 0.5$

6. What is the probability that a card selected at random from a standard deck of 52 cards is an ace or a heart?

$|S| = 52$

4 Aces

52/4 suits = 13 hearts

4 ⇒ Ace

(13) contains 1 case that Ace of hearts

$|E_1| = 4 \Rightarrow p(E_1) = \frac{4}{52}$

$|E_2| = 13 \Rightarrow p(E_2) = \frac{13}{52}$

$p(E_1 \cap E_2) = \frac{1}{52}$

30.77%

$P(E) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= 16/52 = 4/13$

7. What is the probability that when a coin is flipped six times in a row, it lands heads up every time?

$|S| = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6 = 64$

H H H H H H

$|E| = 1 \Rightarrow p(E) = \frac{1}{64} \simeq 1.56\%$

8. What is the probability that a five-card poker hand contains the ace of hearts?

pick 5 cards from a deck of 52 cards ?

$|S| = C(52, 5)$

If 1 card is Ace of heart

1 [2] 3 4 5

Ace of heart

The rest 4 cards can randomly choose from 51 cards $\Rightarrow \boxed{C(51, 4)}$

$|E|$

$$P(E) = \frac{C(51,4)}{C(52,5)} = \frac{\cancel{51!}}{\cancel{47!}\,4!} * \frac{\cancel{47!}\,5!\,5}{52!\,\cancel{52}} = \frac{5}{52} \simeq$$

$\frac{47}{52}$

9.62%

9. What is the probability that a five-card poker hand does not contain the queen of hearts?

$|S| = C(52, 5)$

$|E| = C(51, 5)$

$\Rightarrow P(E) = \frac{C(51,5)}{C(52,5)} =$

$\frac{51!}{46!\,5!} \div \frac{52!}{47!\,5!} \Leftrightarrow \frac{51!}{46!\,5!} \times \frac{47!\,5!}{52!} = \frac{47}{52}$

10. What is the probability that a five-card poker hand contains the two of diamonds and the three of spades?

$|S| = C(52, 5)$, There are 3 cards can be picked from 50 cards

$|E| = C(50, 3)$

$P(E) = \frac{C(50,3)}{C(52,5)} = \frac{50!}{47!\,3!} \times \frac{47!\,5!}{52!} = \frac{20}{51 \cdot 52} = \frac{5}{663}$

$= 0.0075$

11. What is the probability that a five-card poker hand con- tains the two of diamonds, the three of spades, the six of hearts, the ten of clubs, and the king of hearts?

$|S| = C(52,5)$

$|E| = 1$

$P(E) = \frac{1}{C(52,5)} = \frac{1}{\quad}$

12. What is the probability that a five-card poker hand contains exactly one ace?

$|S| = C(52,5)$ we have 4 Aces: $C(4,1) = 4$

The rest 4 cards will be picked from 48

$C(48,4)$ $\quad |E| = 4\,C(48,4)$

$$\text{prob} = \frac{4\,C(48,4)}{C(52,5)} = 4 \cdot \frac{48!}{44!\,4!} \times \frac{47!\,5!}{52!}$$

$45 \cdots 47$ $\quad 5$

$49 \cdots 52$

$$= \frac{4 \cdot 45 \cdot 46 \cdot 47 \cdot 5}{49 \cdot 50 \cdot 51 \cdot 52} = \; ???$$

Conditional Probability

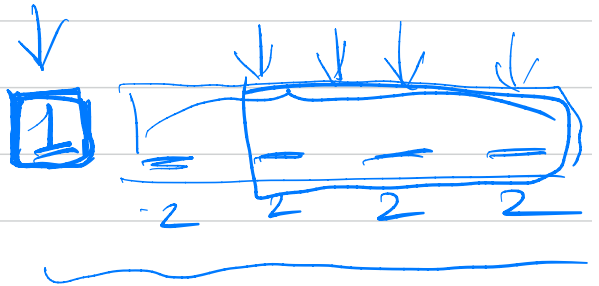
E, F Prob. of E given F is defined by

$$P(E|F) = \frac{P(E \cap F)}{P(F)} \qquad (*)$$

23. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?

F: "first flip comes head"

$P(F) = \frac{2^4}{2^5} = \frac{16}{32} = \frac{1}{2}$



$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2^5}{4}}{2^5} \times \frac{2}{1} = \frac{8}{32} = \frac{1}{4}$

$2^5 = 32$ cases

$P(E \cap E) = \frac{C(4,3)}{2^5} = \frac{4!}{1!\,3!} \times \frac{1}{2^5} = \frac{4}{2^5}$

24. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails?

E

$|S| = 2^5 = 32$

$P(F) = \frac{1}{2}$

$1 = |E \cap F|$: "4 heads out of 5 times flipping, first flip is T

01111

$P(E \cap F) = \frac{1}{32}$

$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/32}{1/2} = \boxed{1/16}$

25. What is the conditional probability that a randomly gen- erated bit string of length four contains at least two con- secutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)

F: "1st bit is 1" $\Rightarrow$ $P(F) = \frac{1}{2}$

$E \cap F = \{1000, 1001, 1100\}$ 3 cases

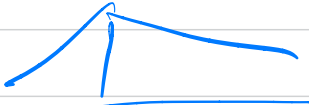
$\Rightarrow$ $P(E \cap F) = \frac{3}{2^4} = \frac{3}{16}$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{3/16}{1/2} = \boxed{\frac{3}{8}}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow \boxed{P(F)\,P(E|F) = P(E \cap F)}$$

Graph — Tree



shortest path

spanning Tree