Homework

- Reading
  - Tokheim, Section 5-1, 5-2, 5-3, 5-7, 5-8
- Machine Projects
  - Continue on MP4
- Labs
  - Continue labs with your assigned section

Designing Logic Circuits

- We want to be able to design a combinational logic circuit from a truth table methodically
  - Sum of Products
  - Product of Sums
- Then we want to be able to simplify it to use the fewest possible gates to implement it
  - Factoring the Boolean logic equation
  - Karnaugh Maps

Maxterm / Minterm
Product of Sums/Sum of Products

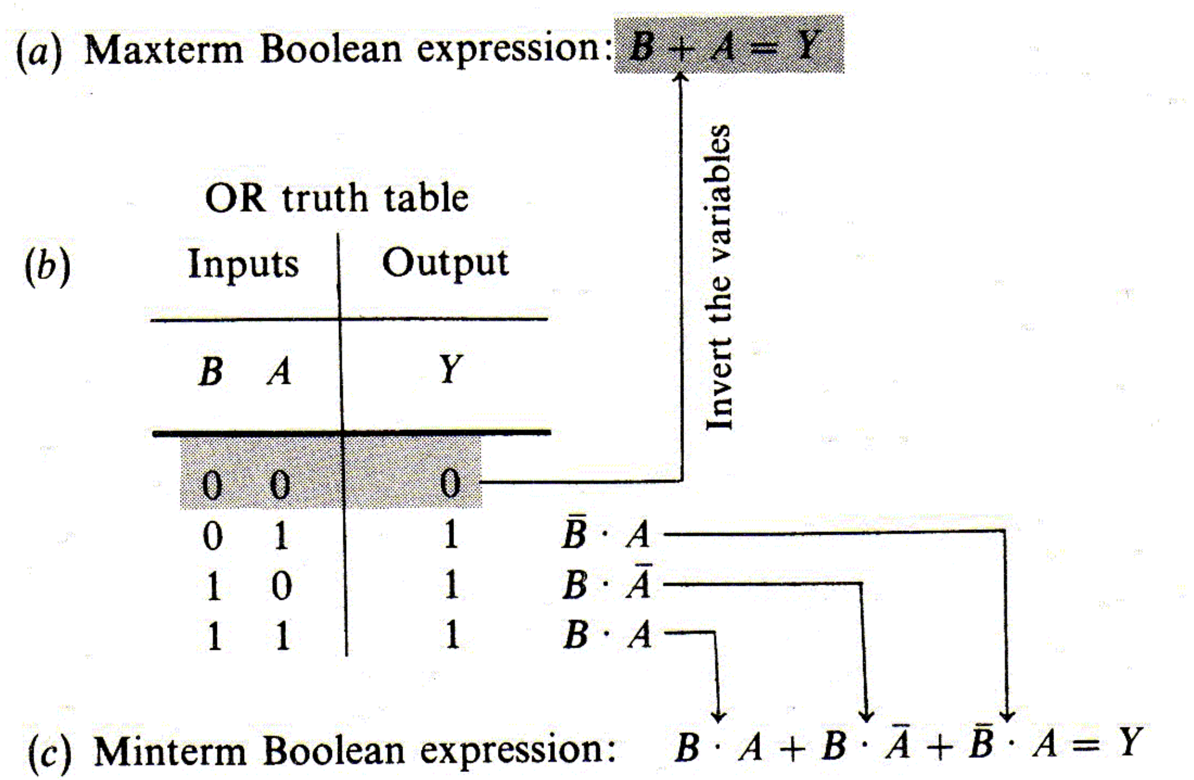


Fig. 5-7

Product of Sums

- Also known as Maxterm expression
- We take each line of the truth table that results in a value of 0 for the output
- We develop a “product” (an AND of each sum term that should create an output value of 0)
- Results in a layer of OR gates followed by an AND gate

Product of Sums

(*a*)

Inputs			Output
C	*B*	*A*	*Y*
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Invert variables

Invert variables

(*b*) Maxterm Boolean expression: $(\bar{C} + \bar{B} + \bar{A}) \cdot (\bar{C} + B + A) = Y$

Fig. 5-8 Developing a maxterm expression

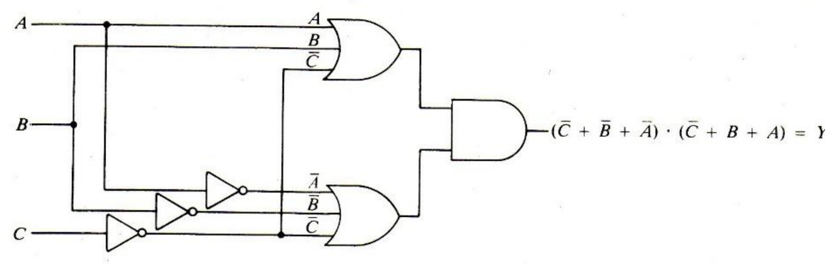


Fig. 5-9 Maxterm expression implemented with OR-AND circuit

Sum of Products

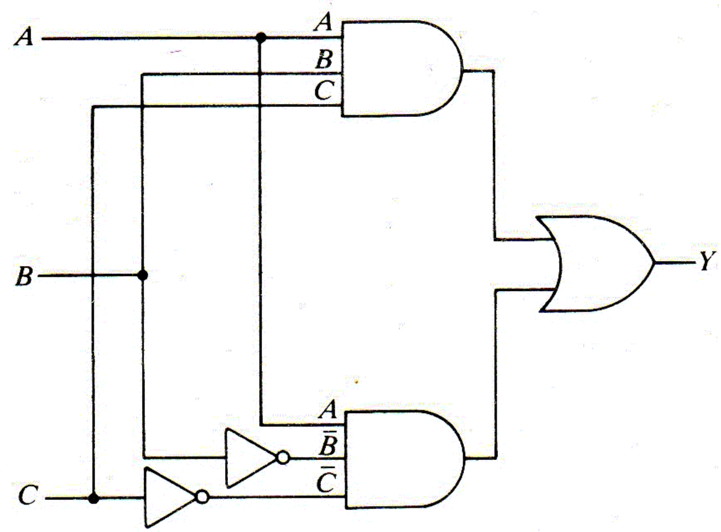
- Also known as Minterm expression
- We take each line of the truth table that results in a value of 1 for the output
- We develop a “sum” (an OR of each product term that should create an output value of 1)
- Results in a layer of AND gates followed by an OR gate

Sum of Products

(*a*)

Inputs			Output	
C	*B*	*A*	*Y*	
0	0	0	0	
0	0	1	1	$\bar{C} \cdot \bar{B} \cdot A$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	1	$C \cdot B \cdot A$

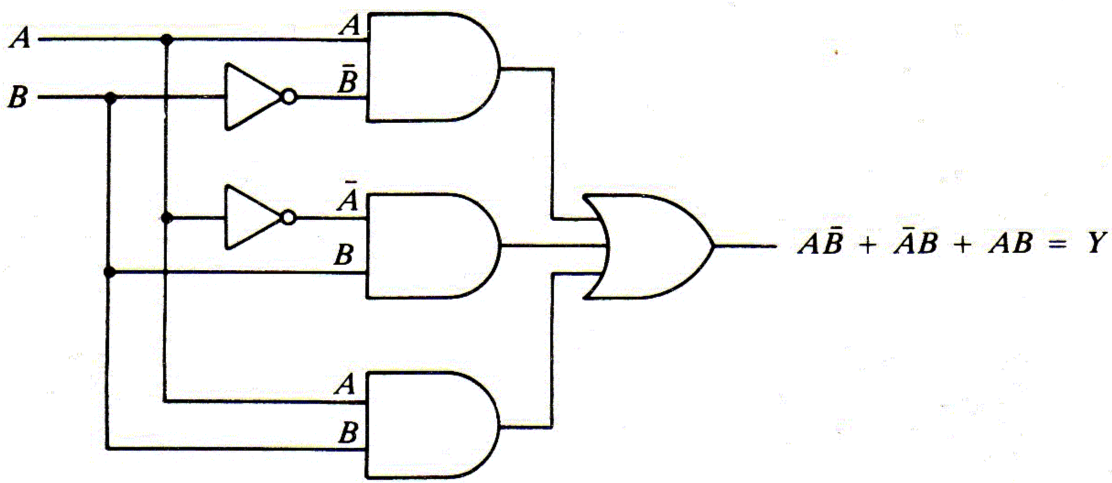
(*b*) Boolean expression: $C \cdot B \cdot A + \bar{C} \cdot \bar{B} \cdot A = Y$



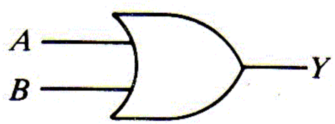
(*c*) Equivalent AND-OR logic circuit

Fig. 5-2

Simplifying Logic Circuits
Minterm / Sum of Products



(*a*) Unsimplified logic circuit



(*b*) Simplified logic circuit

Inputs		Output
B	*A*	*Y*
0	0	0
0	1	1
1	0	1
1	1	1

(*c*) Truth table for OR function

Fig. 5-1

Factoring the Boolean Equation

- Expand the original sum of products:

$$Y = A\overline{B} + \overline{A}B + AB + AB$$
$$= A\overline{B} + AB + \overline{A}B + AB$$

- Factor out A and B from pairs of terms:

$$Y = A(\overline{B} + B) + (\overline{A} + A)B$$
$$= A(1) + (1)B$$
$$= A + B$$

- Not easy to see the steps needed to factor

Karnaugh Maps
Minterm / Sum of Products

- A graphical way to reduce the complexity of a logic equation or truth table
- A tool to bring into play the human ability to recognize patterns
- Draw out the pattern of output 1’s and 0’s in a matrix of input values
- Loop the 1’s and derive product terms to sum
- Notice the order of inputs along edge of matrix

Karnaugh Maps (2 Inputs)

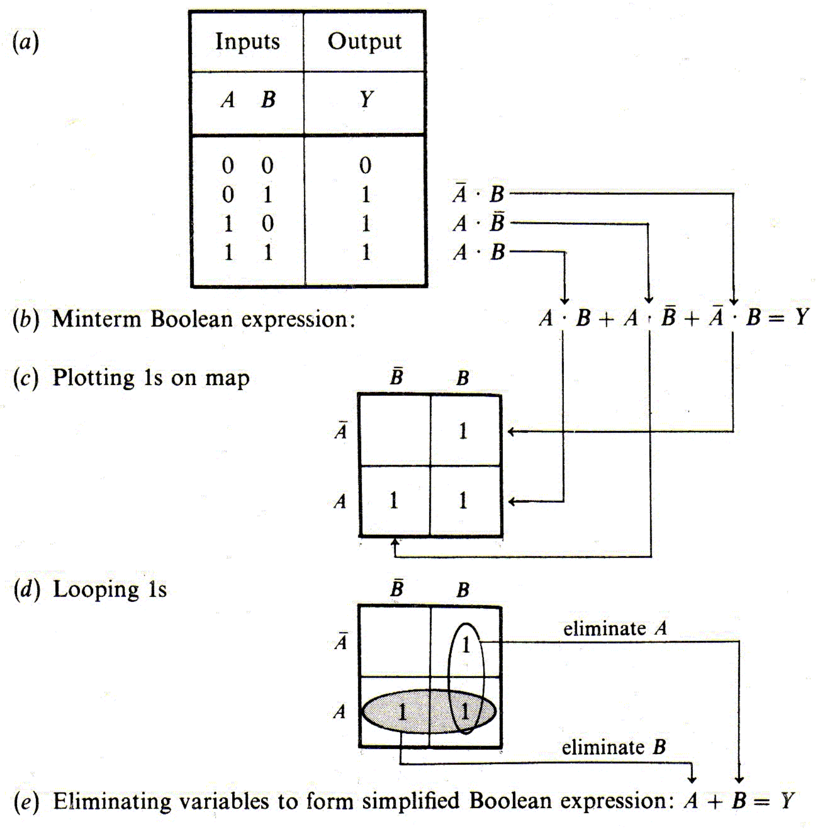


Fig. 5-27 Using a map

Karnaugh Maps (3 Inputs)

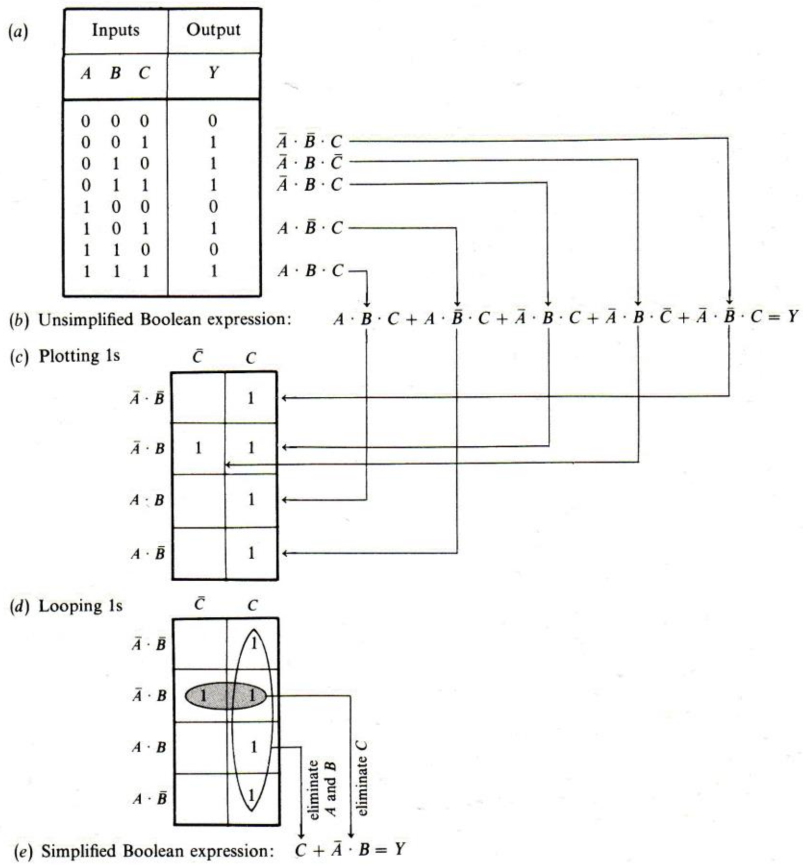


Fig. 5-28 Using a three-variable map

Karnaugh Maps (4 Inputs)

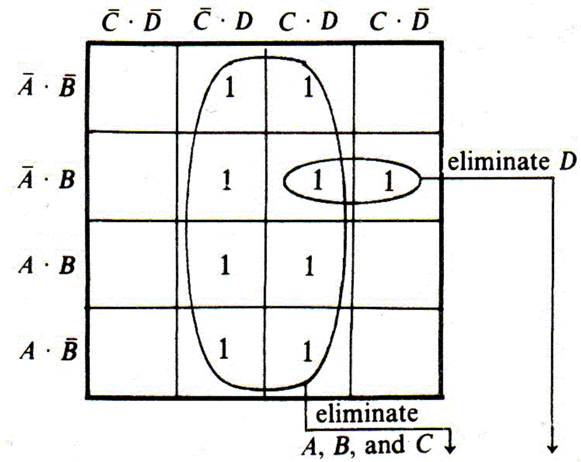
(a)

Inputs				Output
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

(*b*) Unsimplified minterm expression

$$\bar{A}\cdot\bar{B}\cdot\bar{C}\cdot D + \bar{A}\cdot\bar{B}\cdot C\cdot D + \bar{A}\cdot B\cdot\bar{C}\cdot D + \bar{A}\cdot B\cdot C\cdot\bar{D} + \bar{A}\cdot B\cdot C\cdot D + A\cdot\bar{B}\cdot\bar{C}\cdot D + A\cdot\bar{B}\cdot C\cdot D + A\cdot B\cdot\bar{C}\cdot D + A\cdot B\cdot C\cdot D = Y$$

(*c*) Plotting and looping 1s on map



(*d*) Simplified Boolean expression: $D + \bar{A}\cdot B\cdot C = Y$

Fig. 5-31 Using a four-variable map

Karnaugh Maps

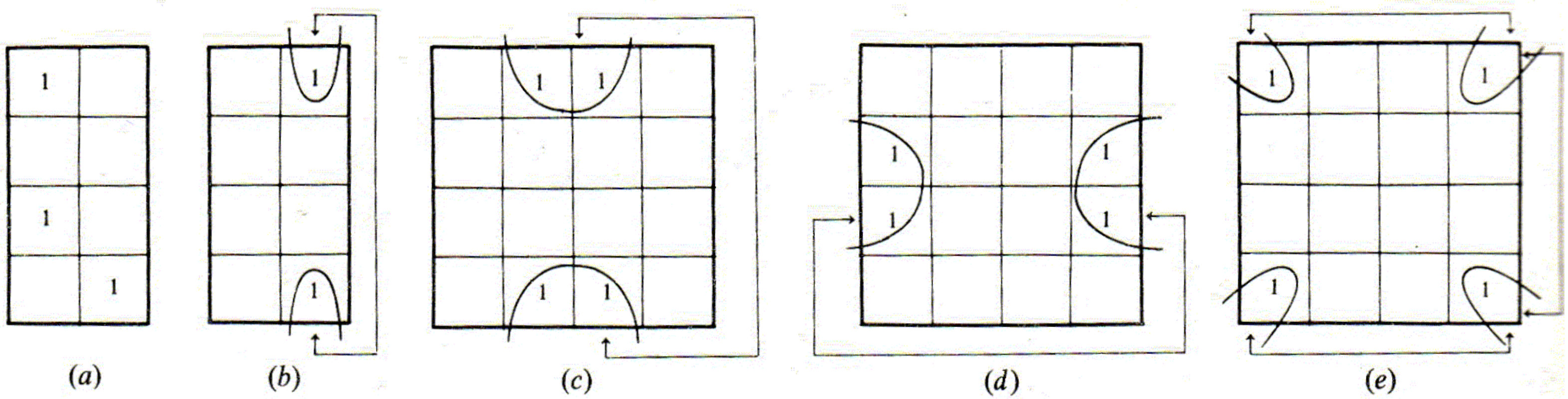


Fig. 5-32 Some unusual looping variations

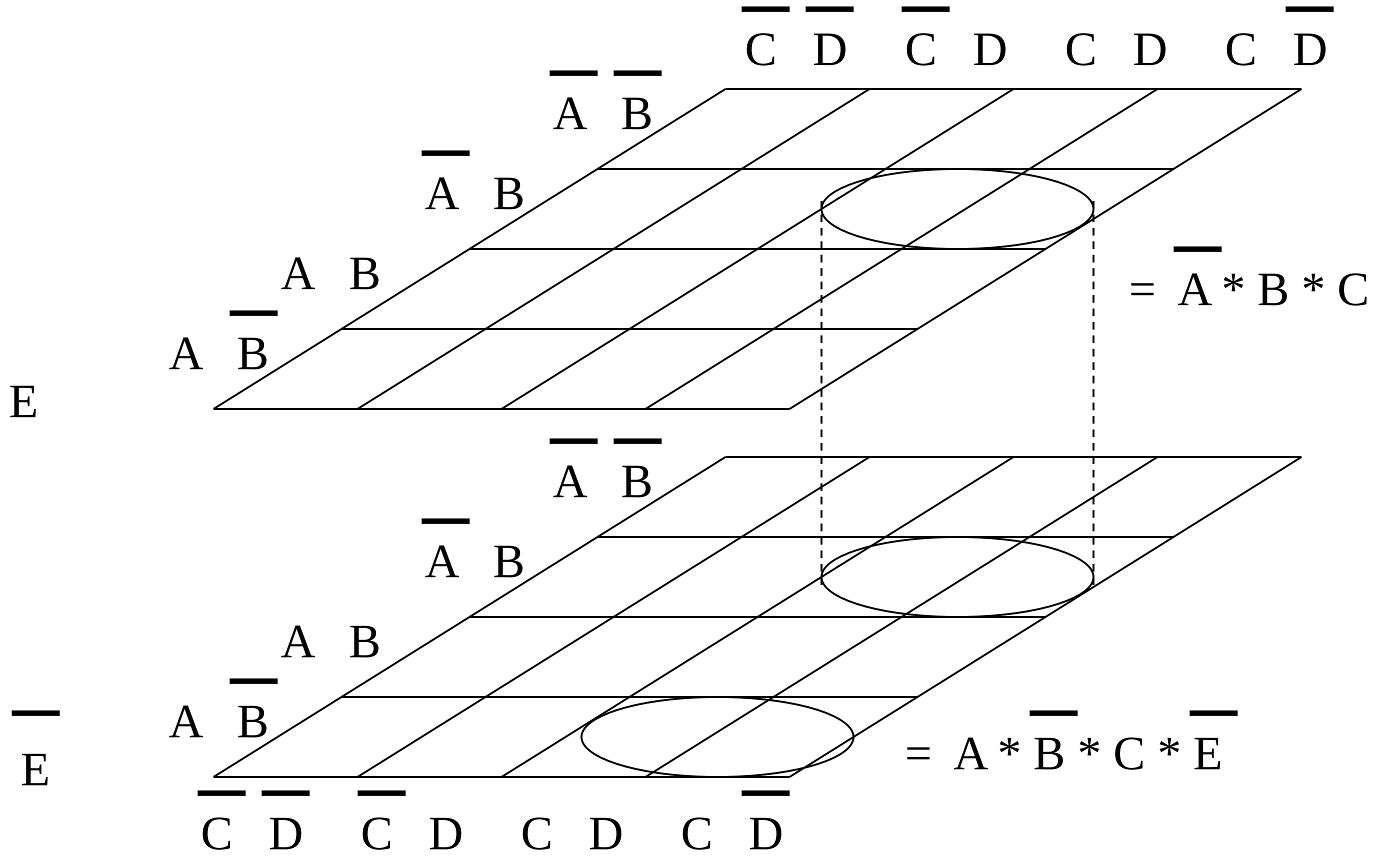
Karnaugh Map Tool

- Link in the references section on my website:
  Free Karnaugh Map Tool
  http://puz.com/sw/karnaugh/
- Let's experiment with it now

Karnaugh Map Blank (4 Input)

	$\overline{C}\,\overline{D}$	$\overline{C}\,D$	$C\,D$	$C\,\overline{D}$
$\overline{A}\,\overline{B}$				
$\overline{A}\,B$				
$A\,B$				
$A\,\overline{B}$				

Karnaugh Map (5 Input)



Karnaugh Map Blank (5 Input)