Resolution & First-Order Logic In-Class-Exercises

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Using Resolution for Propositional Calculus

- Makes a proof by contradiction
- Works with clauses --- knowledge is represented as a conjunction of disjunctions (conjunctive normal form)
- Does not use modus ponens; uses the inference rule of resolution (m,n≥0):

$$\frac{A_1 \vee \ldots \vee A_n \vee \mathbf{C};\quad B_1 \vee \ldots \vee B_m \vee \sim\mathbf{C}}{A_1 \vee \ldots \vee A_n \vee B_1 \vee \ldots \vee B_m}$$

- Steps of a resolution proof:
  1. Convert assumption into clauses
  2. Convert negated conclusion into clauses
  3. Determine if the empty clause can be derived from the clauses generated in steps 1 and 2.
     - Yes: theorem is proven
     - No*: theorem is not proven

Proof by Contradiction

- Assume you want to prove: A1,…,An |- B then the truth of this statement is verified as follows:
  - We assume that A1,..,An is true
  - We assume that ~B is true
  - We show that it can never be the case that A1,..,An are true and B is false…; that is, we look for a contradiction (e.g. P and ~P are both true).

Resolution for Propositional Calculus

Proof by Resolution:

$P \vee (Q \wedge R), Q \rightarrow S, P \rightarrow Q \models R \rightarrow S$

Resolution for Propositional Calculus

Solution: $P \vee (Q \wedge R), Q \rightarrow S, P \rightarrow Q \models R \rightarrow S$

Clauses:

(1) P v Q
(2) P v R
(3) ~Q v S
(4) ~P v Q
(5) R
(6) ~S
(7) ~Q using 3,6
(8) ~P v S using 3,4
(9) ~P using 6,8
(10) Q using 1,9
(11) nil by ~Q $\wedge$ Q using 7,10

Represent the following sentences in first-order logic

- Some students took French in Spring 2001
- Every student who takes French passes it
- Only one student took Greek in spring 2001
- Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they cannot fool all of the people all of the time.