

Semi-supervised based Active Class Selection for Automatic Identification of Sub-Kilometer Craters

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Abstract—Counting craters is a fundamental task of planetary science, because it provides the only tool for measuring relative ages of planetary surfaces. In this paper, we combine active learning with semi-supervised learning to build a new semi-supervised active class selection system for crater detection from high resolution panchromatic planetary images. We propose the Semi-supervised Active Class Selection Algorithm to iteratively enrich an original small training set, without additional human labeling effort, to detect craters from a large volume of images. We propose two strategies to improve detection accuracy by integrating classification with exploration on unlabeled samples. The Majority Vote Strategy is used to automatically obtain class labels by exploiting unlabeled samples from test images. In the same time, the Active Stability Strategy is used to obtain an appropriate class distribution in the constructed training set by detecting unstable classes. By using those two strategies, we actively select test instances from test images into an existing small initial training set while re-learning the classifier in the mean time. The proposed algorithm is empirically evaluated on a large challenging Martian image, exhibiting a heavily cratered Martian terrain characterized by heterogeneous surface morphology. The experimental results demonstrate that the proposed approach achieves a higher accuracy than other existing approaches to a large extent.

I. INTRODUCTION

Craters are among the most studied geomorphic features in the Solar System because they yield information about the past and present geological processes and provide the only tool to measuring relative ages of observed geologic formations. The size distribution of craters conforms to the power-law as large craters are rare and small craters are abundant. Counts of significant number of craters, especially small craters, must be collected from spatially extended regions in order to accumulate sufficient number of samples for accurate statistics. Geologic stratigraphy based on manually collected databases has coarse spatial resolutions. Finer spatial resolutions of the stratigraphy can only be obtained from statistics of smaller craters, and the only viable means to obtain spatially comprehensive databases of small sub-kilometer craters is through automating the process of crater detection.

It becomes extremely challenging to automatically count a very large number of small, sub-kilometer size craters in a deluge of high resolution planetary images. Identification of craters in remotely sensed images can be considered as a special case of object detection in images—an important task in computer vision exemplified by a popular task of face

detection. However, craters have characteristics unlike most objects traditionally subjected to automated identification in images, because they are numerous, have large range of sizes, and they continuously merge into a background. Craters lack specific features that can reliably discriminate them from other objects, or collection of objects, also present on planetary surfaces, including volcanic cones and valley fragments resembling craters.

Supervised learning is one of the approaches that have been used in crater detection (detailed discussion in Section II). Many factors impact the crater detection rate in supervised learning, including feature construction and selection, training set construction, and classifier induction. In this paper, we focus on the problem of training set construction. It is impractical to manually build a comprehensive training set using a large pool of planetary images. We propose the Adaptive Selective Algorithm to iteratively enrich an original small training set without additional human labeling effort to detect craters from a large volume of test images. We propose two strategies to improve detection accuracy from two different perspectives by integrating classification with exploration on unlabeled test samples. The Majority Vote Strategy is used to automatically obtain class labels by exploiting unlabeled samples. The Active Stability Strategy is used to obtain appropriate class distribution in the constructed training set by detecting unstable classes. By using those two strategies, we actively select new instances from test images into existing small initial training set and rebuild the classifier. Our proposed algorithm is empirically evaluated on a large high resolution Martian image, containing 3,500 sub-kilometer craters. The study site presents a challenging case for any crater detection task as it exhibits a heavily cratered Martian terrain of $37,500 \times 56,260$ m^2 , characterized by heterogeneous surface morphology. The experiment results demonstrate that the proposed approach achieves a higher accuracy than other existing approaches to a large extent.

II. RELATED WORK

Existing approaches of detecting craters from planetary images can be divided into two general categories of unsupervised and supervised methods.

The unsupervised methods rely on image processing techniques to identify crater rims in an image as circular or

elliptical features [1], [2]. The original image is preprocessed to enhance the edges of the rims, and edge detection. The performance of unsupervised methods on crater detection is usually worse than that of supervised methods.

The supervised methods [3], [4] use machine learning concepts to build a classifier model from a training set to detect craters. In a learning phase, the training set of images, containing craters labeled by a domain expert, is fed into a learning algorithm. In the detection phase, the previously induced classification model detects craters in a new, unlabeled set of images. In [4], a number of algorithms are tested and the Support Vector Machine (SVM) algorithm is shown to achieve the best rate of crater detection. Most recently, advances in face detection research are incorporated into crater detection techniques. In [5], the combination of edge detection, template matching, and neural network-based false positive recognition scheme is used for detecting craters on Mars. In [6] a boosting algorithm, originally developed by [7] in the context of face detection, is adopted for identification of craters on Mars. In [8], a boosting transfer learning algorithm is used for crater detection.

Active learning [9] has gained much attention recently. In an active learning setting, a classifier is first trained from an initial small training set, and the classifier is used to classify an unlabeled test set. Then, it selects instances from an unlabeled data set, asks a domain expert to label those selected instances, and adds those instances into the training set. The active learning requires a human annotator to label all instances that the algorithm selects. It is still quite time-consuming and expensive thus impractical in crater detection from remotely sensed images. The Self-Training [10] in semi-supervised learning is an approach for building the training set from the unlabeled data set automatically. Self-Training assumes that a classifier's prediction, at least the high confidence ones, tend to be correct. It uses unlabeled data without additional human effort and only selects those with high confidence. It may increase the accuracy on data in small labeled size problems [11], [12].

To the best of our knowledge, neither active learning nor semi-supervised learning has been studied in the field of crater detection by others [13]. Our method combines the active learning and semi-supervised learning to construct different training set according to different test set to achieve higher accuracy. The proposed Semi-supervised Active Class Selection Algorithm in this paper is different from active learning. Active learning still needs additional human effort on labeling, while we obtain the class labels automatically when expanding the original training set. Our method is also different from Self-Training. Self-Training selects the instances with high confidences while our method achieve better detection accuracy using the Major Vote Strategy and Active Stability Strategy to select the instances from the unlabeled data set. Our method combines active learning using the pool-based sampling scenario and semi-supervised learning to construct the training set dynamically according to a different unlabeled test set.

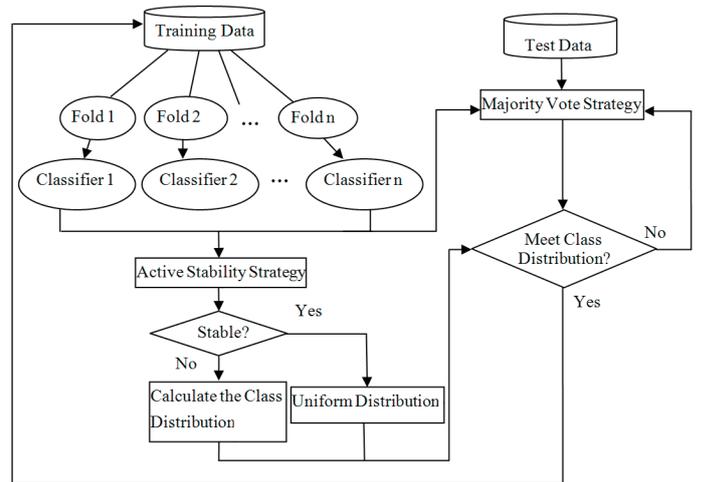


Fig. 1. The flowchart of the Semi-supervised Active Class Selection Algorithm

III. SEMI-SUPERVISED ACTIVE CLASS SELECTION

In this paper, we aim to design and implement a robust classification learning algorithm that is capable of dealing with a large test set when the initial training set is small. In the case of crater detection, it is impractical to ask domain experts to label a large spectrum of craters in many images. Inevitably, an initial training set, which is only generated from a small set of image, may not well represent those in a test set.

We present a new semi-supervised active class selection approach to actively select instances from the test set to the training set to enrich the original small training set. While adding the instances from the unlabeled data set to the training set, we must deal with the following two problems:

- 1) **Class Label Acquisition** : How can we automatically decide class labels while adding unlabeled instances to the training set? Our method does not need a human annotator to label instance on the test set. Instead, the learning algorithm will label the unlabeled instances as craters or non-craters.
- 2) **Active Class Selection** : How many instances should we add with respect to each class, iteratively? In our crater detection case study, we have two classes, craters and non-craters. We need to dynamically decide the class ratio between the crater class and non-craters class while expanding the original training set.

The Semi-supervised Active Class Selection algorithm is designed to solve the problems of class label acquisition and active class selection. Figure 3 depicts the whole process of the algorithm. Specially, our algorithm first partitions the initial training set to n folds and uses a supervised learning algorithm (e.g. SVM) to build n classifiers from the n folds. The algorithm applies the n classifiers to test set and produces n labels for each test instance. Then we use the Majority Vote Strategy (see Section III-A) to obtain the class labels to address the class label acquisition problem. After labeling the instances

in the test set, we use the Active Stability Strategy (see Section III-B) to test whether class distribution in the current training set is stable to address the active class selection problem, if it is stable, we use uniform distribution, if not, we calculate the desired class distribution. Then, the algorithm selects the instances till it satisfies the expected class distribution. The algorithm adds those newly selected test instances into the training set and perform the whole process again until no more qualifying new test instances can be found.

A. Majority Vote Strategy

The Majority Vote Strategy is designed to solve the class label acquisition problem.

Let T be the training set, $T = \{(\vec{x}_i, y_i)\}_{i=1}^t$, where $\vec{x}_i = \langle f_{i,1}, f_{i,2}, \dots, f_{i,m} \rangle$ is the feature vector of instance \vec{x}_i in the training set, y_i is the class label of the instance \vec{x}_i . $y_i \in \{0, 1\}$ for non-crater and crater instances, respectively. t is the number of instance in the training set. Let U be the test dataset, $U = \{\vec{x}_j\}_{j=1}^u$, $\vec{x}_j = \langle f_{j,1}, f_{j,2}, \dots, f_{j,m} \rangle$ be the feature vector of instance \vec{x}_j in the test set, u is the number of instances in the test set. Let $p_{j,f}$ be the predicted label of \vec{x}_j using the classifier C_f , where $p_{j,f} \in \{0, 1\}$ is for non-crater and crater instances, respectively.

The Semi-supervised Active Class Selection Algorithm builds n classifiers $\{C_f\}_{f=1}^n$. The Majority Vote Strategy assigns a class label to a new test instance if the majority votes of a crater candidate indicating it belongs to this class:

$$\frac{\sum_{f=1}^n p_{j,f}}{n} > \varphi_1, j = 1 \dots \mu \quad (\text{III.1})$$

$$\frac{\sum_{f=1}^n p_{j,f}}{n} < \varphi_2, j = 1 \dots \mu \quad (\text{III.2})$$

φ_1 and φ_2 are the user-defined thresholds, and $\varphi_1 \geq \varphi_2$. If the value of $\frac{\sum_{f=1}^n p_{j,f}}{n}$ is greater than φ_1 , then we classify x_j as a crater, if it is smaller than φ_2 , then we classify x_j as a non-crater. If $\varphi_2 \leq \frac{\sum_{f=1}^n p_{j,f}}{n} \leq \varphi_1$, then we do not label this candidate, because the n classifiers are uncertain about the class label of this candidate. And the Majority Vote Strategy will not add this instance to the training set. This strategy carefully selects new instances on which the classifiers have high confidence and avoids those instances on which the existing training set cannot consensus. The higher value of φ_1/φ_2 , less but more strong crater/non-crater examples will be selected. The lower value of φ_1/φ_2 , more but less strong crater/non-crater examples will be selected.

B. Active Stability Strategy

The Active Stability Strategy is designed to solve the active class selection problem.

The idea behind this strategy is that the instances whose predicated class labels changed in this iteration compared to the last iteration are near volatile boundaries. Thus, we assess which classes are near volatile boundaries and sample more instances from those unstable classes in the test set.

The Active Stability Strategy is used only on the training set to obtain the sampling distribution. The strategy uses the n classifiers to predict the instances in the training set which composed by the initial training set and new instances selected from the test set by Majority Vote. For a training example, if the new generated class label is different from the previous class label which is generated by the previous n classifiers using the training in last iteration, then it indicates that we need to sample from these unstable classes. We use equation (III.3) to obtain the sampling distribution.

$$Num(c) = \frac{\frac{unstable[c]}{n_c}}{\sum_{i=0}^{|classes|} \frac{unstable[i]}{n_c}} * \rho \quad (\text{III.3})$$

where $Num(c)$ is the number of class c need to be added, $unstable[c]$ is the number of instances whose classification changes, n_c is the number of class c in the training set. We divide $unstable[c]$ by n_c to keep small classes from being ignored and large classes from being over-emphasized. $|classes|$ is the number of classes. ρ is the total number of instances to be added in the round.

[14] proposes an active class selection algorithm named Redistricting which iteratively builds training sets using the labeled data. The major differences between Redistricting and the Active Stability Strategy is that Redistricting builds the training set using the labeled data, while the Active Stability Strategy builds the training set iteratively using the unlabeled data.

C. Semi-supervised Active Class Selection Algorithm

The pseudocode of the Semi-supervised Active Class Selection Algorithm is shown in Algorithm 1.

As depicted in the algorithm, we begin with a Cross Validation with n folds over T_1 , the initial training set. We obtain a prediction for each $\vec{x}_i \in T_1$. In the second round, we collect T_2 of size ρ . We next perform a Cross Validation over all of the data in the training set and create a classifier for each fold. Note that on subsequent iterations, we keep the data from T_{r-1} in the same folds, and stratify only the newly generated data T_{add} into the existing folds. For each fold, we compare the classification results of $P_r(\vec{x}_i)$ and $P_{r-1}(\vec{x}_i)$ on each instance $\vec{x}_i \in T_1$. If the labels are different, then the counter for the class specified by y_i , $unstable[y_i]$, is incremented. We conclude by generating predictions of the new batch of data T_{add} and the increment r . After the second round we add instances using the formula III.3, where c is a class from the set of all classes in the dataset. Steps 8 to 13 use the Majority Vote Strategy to obtain the class labels. Steps 15 to 38 use the Active Stability Strategy. Steps 32 to 37 are used to get the $unstable[c]$. we add the newly generated folds to each fold in the last round $r - 1$ and build new classifier from $\{T_r - T_{r,f}\}$ and then compare the predicted label in this round and the previous round, if it is different. Then $unstable[y_i]$ is increased, y_i is the real class label of instance x_i if it is from the initial training set; otherwise y_i is the predicted label obtained by the Majority

Vote Strategy. The stopping criterion is that no more qualified instances can be added in T_{add} training set.

Algorithm 1 Semi-supervised Active Class Selection

Input: (1) Training set $T = \{(\vec{x}_i, y_i)\}_{i=1}^t$, test set $U = \{\vec{x}_j\}_{j=1}^\mu$.
(2) ρ , number of instances to be added in round r , user defined thresholds
 φ_1, φ_2 and μ .

- 1 Initially, let $T_1 = T = \{(\vec{x}_i, y_i)\}_{i=1}^t, U = \{\vec{x}_j\}_{j=1}^\mu$ and $r = 1$.
- 2 Divide T_1 into n stratified folds $T_{1,1}, T_{1,2}, \dots, T_{1,n}$
- 3 **For** $f = 1$ to n do
- 4 Build Classifier C_f from $\{T_1 - T_{1,f}\}, C_{f'}$ from $T_{1,f}$
- 5 **For** all \vec{x}_i in $T_{1,f}$ do label \vec{x}_i with $P_r(\vec{x}_i)$ with C_f
 end For
- 6 **For** all \vec{x}_j in U , do label \vec{x}_j as $p_{j,f}$ with $C_{f'}$ **end For**
- 7 **end For**
- 8 **If** $\frac{\sum_{f=1}^n p_{j,f}}{n} > \varphi_1, j = 1, \dots, \mu$
- 9 do label $\vec{x}_j = 1$ (1 is for crater)
- 10 **end If**
- 11 **If** $\frac{\sum_{f=1}^n p_{j,f}}{n} < \varphi_2, j = 1, \dots, \mu$
- 12 do label $\vec{x}_j = 0$ (0 is for non-crater)
- 13 **end If**
- 14 **While** no instance can be added in T_{add}
- 15 **If** $r = 2$ then
- 16 $T_{add} =$ random sample size of ρ
- 17 **else**
- 18 Compute $Num(c) = \frac{unstable[c]}{\sum_{i=0}^{|classes|} \frac{n_c \cdot unstable[i]}{n_c}} * \rho$
- 19 Initialize $counts(c), c \in \{0, 1\}$
- 20 **For** all x_j
- 21 **If** $counts(c) < Num(c)$
- 22 $y_i = P(\vec{x}_j)$
- 23 add (\vec{x}_j, y_j) to T_{add}
- 24 $counts(c)++$;
- 25 **end If**
- 26 **end For**
- 27 $T_r = T_{r-1} + T_{add}$
- 28 Initialize $unstable[c], c \in \{0, 1\}$
- 29 Divide T_{add} into n stratified folds
 $T_{add,1}, T_{add,2}, \dots, T_{add,n}$
- 30 **For** $f=1$ to n do
- 31 $T_{r,f} = T_{r-1,f} \cup T_{add,f}$
- 32 Build the Classifier C_f form $\{T_r - T_{r,f}\}$
- 33 **For** all \vec{x}_i in $T_{r,f}$ do label \vec{x}_i as $p_r(\vec{x}_i)$ with C_f **end For**
 For
- 34 **If** $P_r(\vec{x}_i) \neq p_{r-1}(\vec{x}_i)$ unstable $[y_i]++$ **end If**
- 35 **end For**
- 36 $r++$
- 37 **end While**

IV. EXPERIMENTAL RESULTS

We have selected a portion of the High Resolution Stereo Camera (HRSC) nadir panchromatic image h0905 [15], taken by the Mars Express spacecraft, to serve as the case study site for crater detection. As illustrated in Figure 2, the selected image has the resolution of 12.5 meters/pixel and the size of 13,500,000 (3,000 by 4,500) pixels. A domain expert manually marked 3,500 craters in this image to be used as the ground truth to which the results of auto-detection are compared. The image represents a significant challenge to automatic crater detection algorithms. It covers terrain having spatially variable morphology and its contrast is rather poor (this is most noticeable when the image is inspected at a small spatial scale). The central row is characterized by surface morphology that is distinct from the rest of the image. The top and bottom rows have similar morphology but the bottom is much more heavily cratered than the top row.

We identify 12,542 crater candidates in the image using the pipeline depicted in [8]. We totally extract 1,089 Haar-like features and use the feature selection algorithm described in [8] to select top 10 features from the 1,089 features and only those 10 best features are used in the experiments. The training set consists of instances selected randomly from crater candidates located in the 2nd column of the top row in Figure 2. Figure 2 shows the detection result on the case study site. The craters marked by the red rectangles are craters detected by our algorithm. We detect craters larger than 16 pixels (200 m in the H0905 image) because those correspondent candidates can be reliably identified by the method describe in [8]. We are not interested in craters that is larger than 400 pixels (5,000 m in the H0905 image) because those large craters have already been identified. We set thresholds $\varphi_1 = \varphi_2 = 0.5$ for the Majority Vote Strategy and ρ is set to 80 for each iteration for the Active Stability Strategy based on empirical observation. The training set consists of 204 positive instances and 39 negative instances.

Sub-kilometer craters marked by a domain expert are served as the ground truth in our performance evaluation process. The Semi-supervised Active Class Selection Algorithm classifies crater candidates as craters and non-craters. The number of ground truths covered by the craters detected by the algorithm is TP (True Positives). The number of crater candidates that are not craters but mistakenly classified as craters is FP (False Positives). The number of crater candidates that are not craters and correctly classified as non-craters is TN (True Negatives). The number of crater candidates that are craters but failed to be identified as craters is FN (False Negatives). We compare accuracy ($accuracy = \frac{TP+TN}{TP+TN+FP+FN}$) between the Semi-supervised Active Class Selection Algorithm and other three algorithms of random sampling, Redistricting and Self-Training.

- Random Sampling: it is a widely used approach for training data selection. It produces subset of the data which has a distribution similar to the original test data set by randomly sample each instance

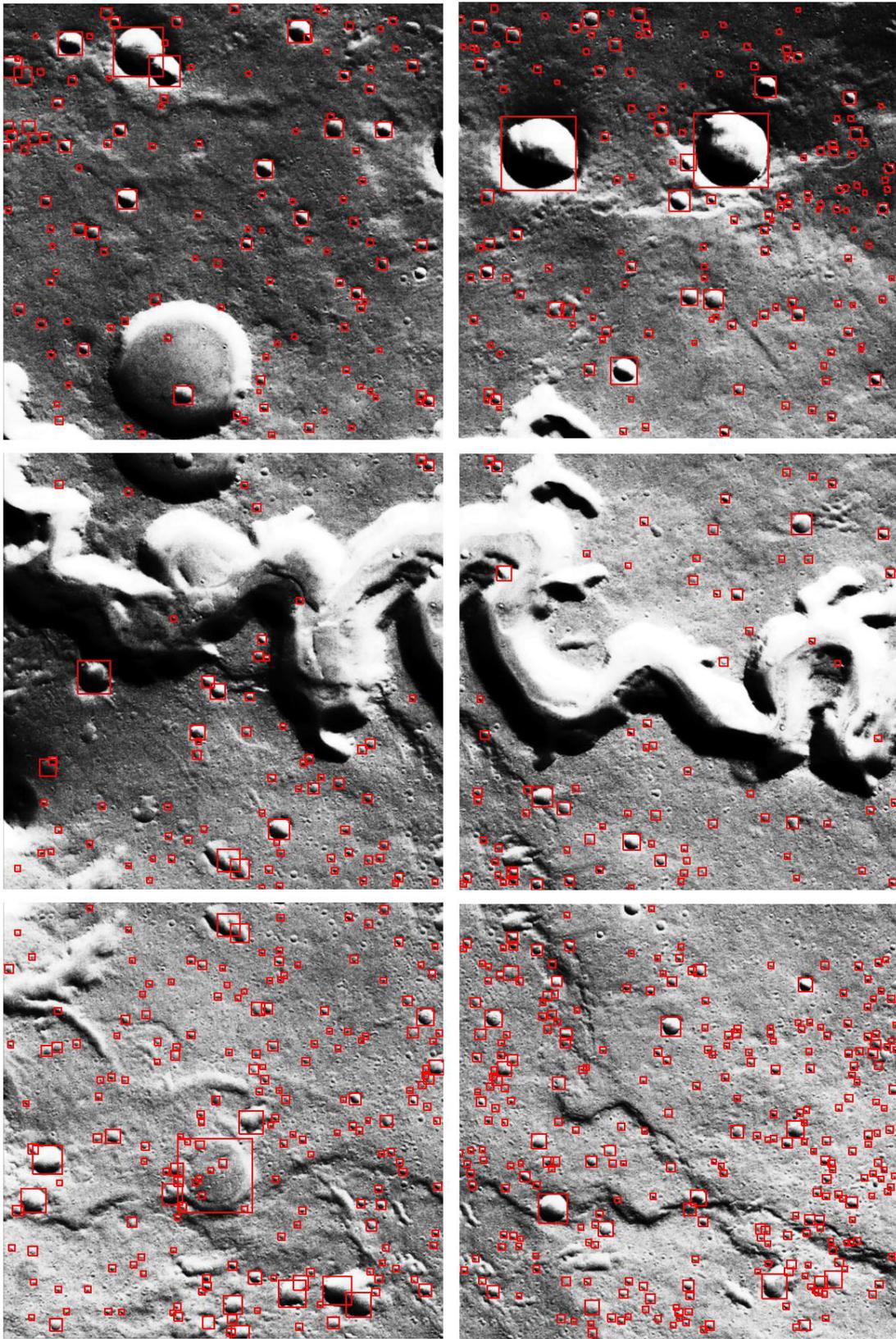


Fig. 2. Detection result on the case study site. The craters marked by the red rectangles are craters detected by our algorithms which only targets at craters that are between 16-pixel and 400-pixel in diameters. Site for the case study, located in the Xanthe Terra, centered on Nanedi Vallis and covers mostly Noachian terrain on Mars, from the image HRSC nadir panchromatic h0905, resolution of 12.5m/pixel. Images, ground truth and detection results can be found at http://kdl.cs.umb.edu/share/detection_result_ISPA/

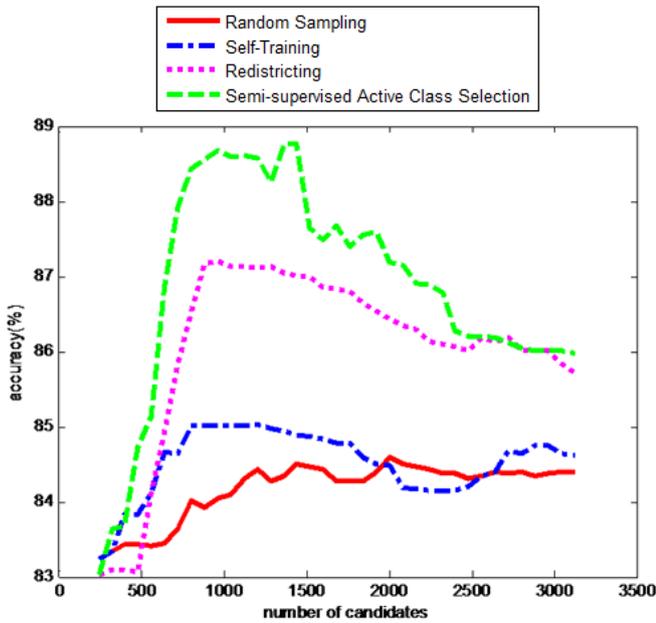


Fig. 3. Performance results of the Random Sampling, Self-Training, Redistricting, and Semi-supervised Active Class Selection.

independently.

- Redistricting: The original method was proposed in [14] and it used the idea of assessing the boundary classes. In our experiments, we implemented the core method of redistricting, that is, it assesses the unstable class and decides the class ratio to be achieved. To assure a fair comparison, the redistricting implemented in our paper does not use the true labels provided by a domain expert. Instead, we obtain the class labels by a classifier trained on the training set which is a typical way in a semi-supervised learning method.
- Self-Training: it adds new instances with high confidence from the test set to the training set.

Figure 3 shows the accuracy using Semi-supervised Active Class Selection, Random Sampling, Redistricting and Self-Training. The ground truth of the entire image serves as an external criterion to evaluate the performance of the four algorithms on the test set. Of the four algorithms, the base classifier used is LIBSVM [16], a SVM classifier using the radial basis function kernel. The experiments have been performed ten times and the average accuracies are reported. From Figure 6, we can see the best accuracy got by Random Sampling, Self-Training, Redistricting and Semi-supervised Active Class Selection are 84.8%, 85.1%, 87.2%, 88.7%. Our Semi-supervised Active Class Selection yields the best accuracy of all the method.

V. CONCLUSIONS AND FUTURE WORK

This paper aims at improving detection rate for auto-detection of small craters in high resolution images of planetary surfaces. The proposed Semi-supervised Active Class

Selection uses innovative methods on training set construction, using active learning and semi-supervised learning. Significant performance gain has been observed in our case study site on Mars.

In future work, we should make analysis on the initial training set. Analyze the instances in the initial training set when transfer the knowledge from the test set and discard the instances which are not compatible with the test set. In addition, how to intelligently decide the thresholds for Majority Vote Strategy should be further studied.

VI. ACKNOWLEDGMENTS

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