

# Homework 1

*Posted: September 21, 2022*

*Due: , October 5, 2022*

1. Give an example of two languages  $L, K$  over the alphabet  $A = \{a, b\}$  such that  $LK = KL$ ; also, give an example of two such languages where  $LK \neq KL$ .
2. Let  $A$  be an alphabet such that  $a, b, c \in A$ . Prove that there are no words  $x, y, z \in A^*$  such that  $xaybz = ybxcz$ .
3. Let  $x, y \in A^*$  be two words such that  $xyy = yxx$ . Prove that  $x = y$ .
4. Let  $L = \{a\}^*\{b\}^+$ . Compute  $x^{-1}L$  for  $x \in \{a, b, ab, ba\}$ .
5. Let  $L = \{x, y\}$  be a language,  $L \subseteq \{a, b\}^*$ . Prove that if  $L^2$  has fewer than 4 words, then there exists a word  $z$  and  $m, n \in \mathbb{N}$  such that  $x = z^m$  and  $y = z^n$ .
6. Let  $u, v$  be words,  $u, v \in A^*$ . Prove that

$$(u^{-1}L)v^{-1} = u^{-1}(Lv^{-1}).$$