Homework 1

Posted: February 9, 2026
Due: February 23, 2026

1. Give an example of two languages L, K over the alphabet $A = \{a, b\}$ such that $LK = KL$; also, give an example of two such languages where $LK \neq KL$.

2. Let A be an alphabet such that $a, b, c \in A$. Prove that there are no words $x, y, z \in A^*$ such that $xaybz = ybxcz$.

3. Let $x, y \in A^*$ be two words such that $xyy = yxx$. Prove that $x = y$.

4. Let $L = \{a\}^*\{b\}^+$. Compute $x^{-1}L$ for $x \in \{a, b, ab, ba\}$.

5. Let $L = \{x, y\}$ be a language, $L \subseteq \{a, b\}^*$. Prove that if L^2 has fewer than 4 words, then there exists a word z and $m, n \in \mathbb{N}$ such that $x = z^m$ and $y = z^n$.

6. Let u, v be words, $u, v \in A^*$. Prove that

$$(u^{-1}L)v^{-1} = u^{-1}(Lv^{-1}).$$