Homework 2

Posted: October 7, 2024
Due: October 21, 2024

1. Show that the class of all total functions is a PRC class.

2. Let f be the function defined as

$$f(x) = \begin{cases} 2x & \text{if } x \text{ is a perfect square,} \\ 2x+1 & \text{otherwise.} \end{cases}$$

   Prove that f is primitive recursive.

3. Let $\sigma(x)$ be the sum of the divisors of x if $x \neq 0$ and $\sigma(0) = 0$. For example, $\sigma(6) = 1 + 2 + 3 + 6 = 12$.

   Prove that $\sigma(x)$ is primitive recursive.

4. Define the predicate SQSM to be TRUE if x is the sum of two perfect squares. For example, SQSM(29) is TRUE because $29 = 2^2 + 5^2$. Prove that SQSM(29) is primitive recursive.

5. Let $F(0) = 0$, $F(1) = 1$, $F(n+2) = F(n+1) + F(n)$. Note that $F(n)$ is the n^{th} Fibonacci number. Prove that $F(n)$ is primitive recursive.

   Hint: Consider the function $G(n) = \langle F(n), F(n+1) \rangle$ for $n \in \mathbb{N}$.