Homework 3

Posted: March 24, 2025
Due: April 12, 2025

1. Let $A = \{a, b\}$ be an alphabet. Compute the minimal dfa capable of recognizing the language A^*bA^+a.

2. Prove that the language $\{a^n b^{10n} c^{20n} \mid n \in \mathbb{N}\}$ is not regular.

3. Let $G = (\{S, X, Y, Z\}, \{a, b\}, S, \{S \to XYZ, X \to SYZ, Y \to SXZ, X \to a, Y \to b, Z \to a\})$ be a context-free grammar. Prove that if $x \in L(G)$, the length of x has the form $3 + 2k$, where $k \geq 0$.

   Hint: Use induction on the length of the derivation $S \overset{*}{\underset{G}{\Rightarrow}} x$.

4. Give an example of a non-regular language such that $\mathsf{PREF}(L)$, $\mathsf{SUFF}(L)$, and $\mathsf{INFIX}(L)$ are all regular languages.