

Homework 4

Posted: April 12, 2025

Due: April 30, 2022 at 4:00pm

1. Let $G = (A_N, A_T, S_0, P)$ be a context-free grammar, where $A_N = \{S_0, S_1, S_2\}$, $A_T = \{a, b\}$, and P contains the following productions:

$$\begin{aligned} S_0 &\rightarrow aS_2, S_0 \rightarrow bS_1, S_1 \rightarrow a, S_1 \rightarrow aS_0, \\ S_1 &\rightarrow bS_1S_1, S_2 \rightarrow b, S_2 \rightarrow bS_0, S_2 \rightarrow aS_2S_2. \end{aligned}$$

Prove that the word $w = abbaab$ is in the language generated by this grammar by constructing a leftmost derivation $S_0 \xrightarrow[G]{*} w$. Draw the derivation tree for w .

2. Prove or disprove the following statements:
 - (a) If L is a context-free language and $K \subseteq L$, then K is a context-free language.
 - (b) If both L and L' are context-free languages on an alphabet A , then $L - L'$ is a context-free language.
 - (c) If L is a context-free language, then LL^R is a context-free language.
 - (d) If LL^R is a context-free language, then L is a context-free language.
3. Find an equivalent context-free grammar G' in Chomsky normal form for the following context-free grammars:
 - (a) $G = (\{S\}, \{a, b\}, S, \{S \rightarrow a, S \rightarrow aS, S \rightarrow aSbS\})$;
 - (b) $G = (\{S, X, Y\}, \{a, b\}, S, \{S \rightarrow XYX, S \rightarrow ab, X \rightarrow SYS, X \rightarrow ba, Y \rightarrow XSX, Y \rightarrow b\})$;

(c) $G = (\{S\}, \{+, *, (,), a\}, S, \{S \rightarrow S + S, S \rightarrow S * S, S \rightarrow a, S \rightarrow (S)\})$.

4. Prove that the following languages are not context-free:

(a) $\{a^n b^{n^2} \mid n \in \mathbb{N}\}$;

(b) $\{a^n b^{2n} c^n \mid n \in \mathbb{N}\}$.

5. Determine whether the following languages are

(i) regular,

(ii) context-free, but not regular,

(iii) not context-free.

(a) $\{xyx \mid x, y \in \{a, b\}^*\}$;

(b) $\{xyx \mid x, y \in \{a, b\}^* \text{ and } |x| = 5\}$;

(c) $\{xyx^R \mid x, y \in \{a, b\}^*\}$;

(d) $\{xyx^R \mid x, y \in \{a, b\}^* \text{ and } |x| = 5\}$.