Homework 5

Posted: April 30, 2025
Due: May 14, 2025

1. Prove that if G is a context-free grammar in Chomsky normal form, then for any $x \in L(G)$ with $|x| = n$ the derivation of x requires $2n - 1$ steps.

2. Prove that the language

$$L_0 = \{a^n b^m c^p \mid n, m, p \in \mathbb{N}, n < m\}$$

over the alphabet $A = \{a, b, c\}$ is context free by constructing the corresponding pushdown automaton.

3. Let $\mathcal{M}$ be a pda and let $L \subseteq A^*$ be a language such that $L = L(\mathcal{M})$. Prove that if the length of the stack never exceeds a fixed number $k \in \mathbb{N}$, then L is a regular language.

4. Let $L = \{a^{3n} b^{5n} \mid n \in \mathbb{N}\}$. Prove that L is a deterministic context-free language by constructing a dpda $\mathcal{M}$ such that $L = L(\mathcal{M})$.