Wavelets and Applications

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The Haar Transform

Values of an analog signal measured at time values $1, \ldots, n$ are $x_1, \ldots, x_n$.
The *support* of the sequence **x** $= (x_1, \ldots, x_n)$ is the set $\{i | x_i \neq 0\}$.
We form two sequences of size $n/2$:

$$t_1, \ldots, t_{n/2}, \text{ and } f_1, \ldots, f_{n/2}$$

$$t_m = \frac{x_{2m-1} + x_{2m}}{\sqrt{2}} \text{ and } f_m = \frac{x_{2m-1} - x_{2m}}{\sqrt{2}}$$

for $1 \leq m \leq \frac{n}{2}$.

Example:

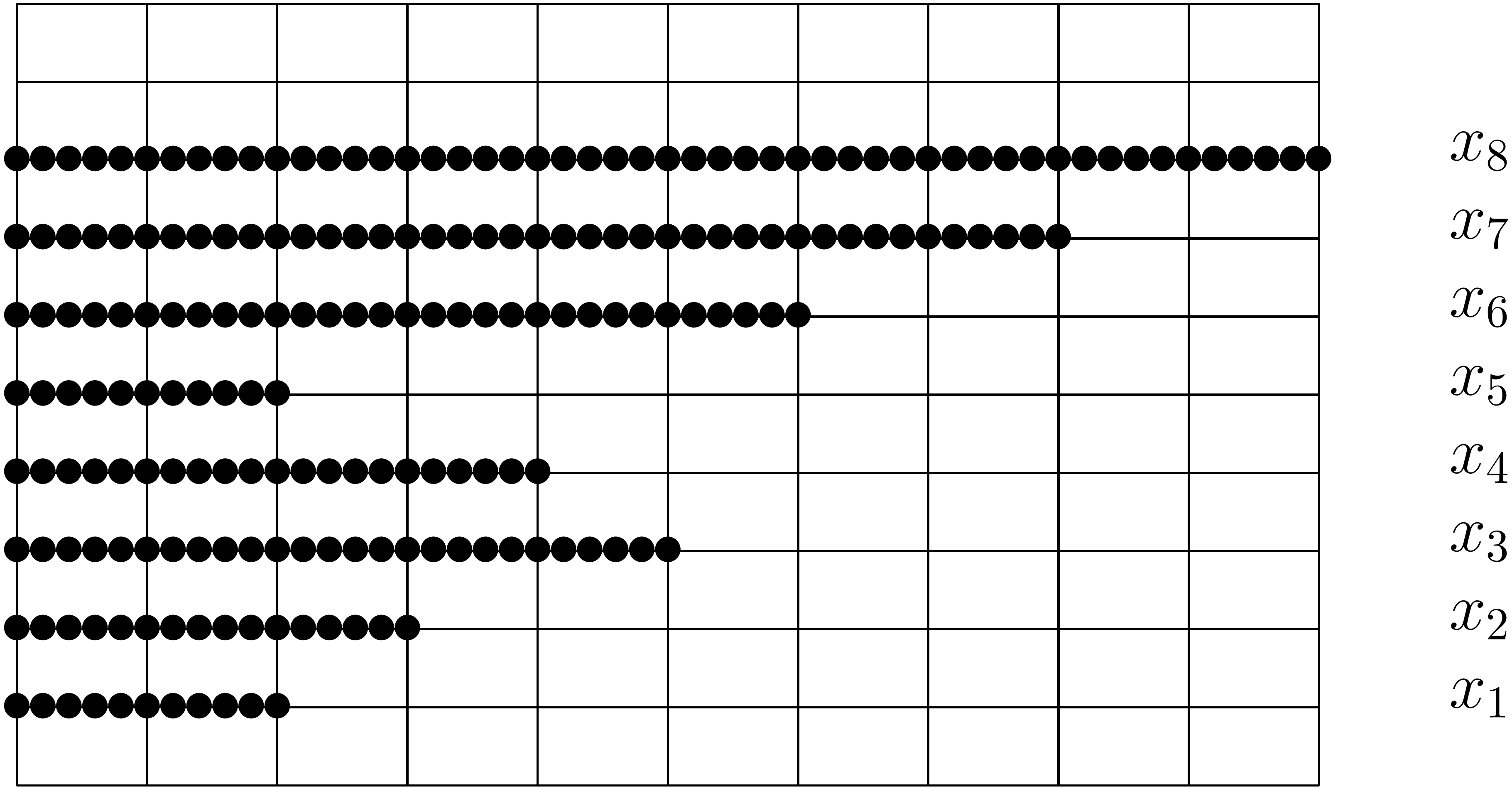
Let

$$\mathbf{x} = (x_1, \ldots, x_8) = (2, 3, 5, 4, 2, 6, 8, 10)$$

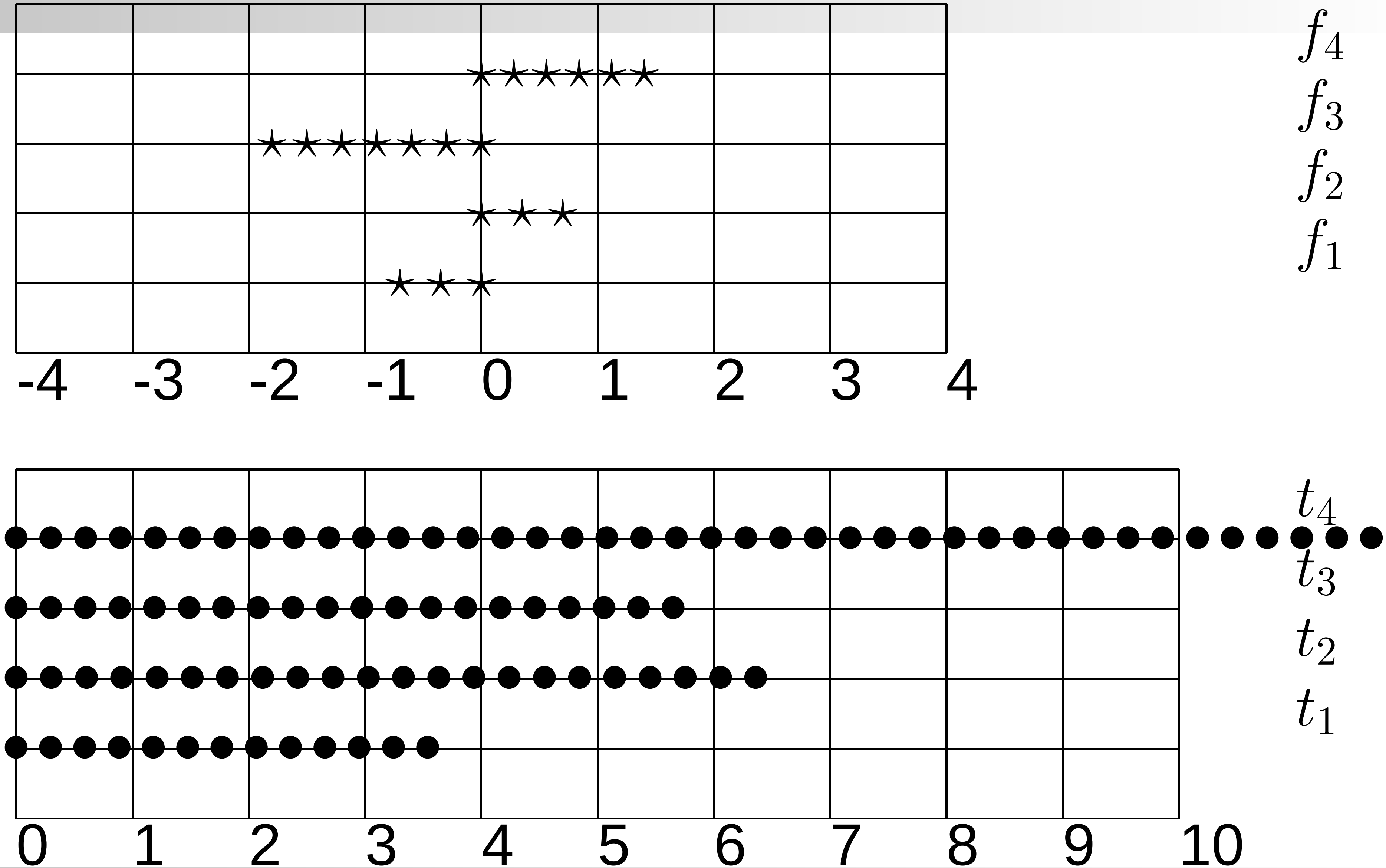
$\mathbf{t} = (t_1, \ldots, t_4)$ and $\mathbf{f} = (f_1, \ldots, f_4)$:

$$\begin{aligned} t_1 &= \frac{5}{\sqrt{2}} = 3.54 & f_1 &= \frac{-1}{\sqrt{2}} = -0.70 \\ t_2 &= \frac{9}{\sqrt{2}} = 6.36 & f_2 &= \frac{1}{\sqrt{2}} = 0.70 \\ t_3 &= \frac{8}{\sqrt{2}} = 5.65 & f_3 &= \frac{-4}{\sqrt{2}} = -2.80 \\ t_4 &= \frac{18}{\sqrt{2}} = 12.85 & f_4 &= \frac{-2}{\sqrt{2}} = -1.40 \end{aligned}$$

Original Sequence $x_1, \ldots, x_8$



Fluctuations and Trends



Remarks...

- The trend components $t_1, \ldots, t_4$ approximate the trends in **x**.
- The fluctuation components $f_1, \ldots, f_4$ approximate the fluctuations of **x**.
- Conservation of energy:

$$\mathcal{E}(\mathbf{x}) = \sum_{i=1}^{8} x_i^2 = \sum_{i=1}^{4} t_i^2 + \sum_{i=1}^{4} f_i^2,$$

- Fluctuations are small because **x** originates typically in sampling of a continuous signal.

Haar Transform

The Haar transform is the mapping $\mathcal{H}$: Seq$(\mathbb{R}) \longrightarrow$ Seq$(\mathbb{R})$ given by:

$$\mathcal{H}(x_1, \ldots, x_n) = (t_1, \ldots, t_{\frac{n}{2}}, f_1, \ldots, f_{\frac{n}{2}})$$

for $(x_1, \ldots, x_n) \in$ Seq$(\mathbb{R})$.
We use the condensed notation $\mathcal{H}(\mathbf{x}) = (\mathbf{t}^1 | \mathbf{f}^1)$, where

$$\begin{aligned} \mathbf{t}^1 &= (t_1, \ldots, t_{\frac{n}{2}}) \\ \mathbf{f}^1 &= (f_1, \ldots, f_{\frac{n}{2}}) \end{aligned}$$

The Inverse Haar Transform

If $\mathcal{H}(x_1, \ldots, x_n) = (t_1, \ldots, t_{\frac{n}{2}}, f_1, \ldots, f_{\frac{n}{2}})$, then

$$
\begin{aligned}
t_1 &= \tfrac{x_1+x_2}{\sqrt{2}} & f_1 &= \tfrac{x_1-x_2}{\sqrt{2}} \\
&\vdots & &\vdots \\
t_{\frac{n}{2}} &= \tfrac{x_{n-1}+x_n}{\sqrt{2}} & f_{\frac{n}{2}} &= \tfrac{x_{n-1}-x_n}{\sqrt{2}}
\end{aligned}
$$

Then:

$$
\begin{aligned}
x_1 &= \tfrac{t_1+f_1}{\sqrt{2}} & x_2 &= \tfrac{t_1-f_1}{\sqrt{2}} \\
&\vdots & &\vdots \\
x_{n-1} &= \tfrac{t_{\frac{n}{2}}+f_{\frac{n}{2}}}{\sqrt{2}} & x_n &= \tfrac{t_{\frac{n}{2}}-f_{\frac{n}{2}}}{\sqrt{2}}
\end{aligned}
$$

The Inverse Haar Transform (cont)

The inverse Haar transform is the mapping $\mathcal{H}^{-1}$: Seq$(\mathbb{R}) \longrightarrow$ Seq$(\mathbb{R})$ given by:

$$\mathcal{H}^{-1}(t_1, \ldots, t_{\frac{n}{2}}, f_1, \ldots, f_{\frac{n}{2}}) = (x_1, \ldots, x_n)$$

for $(t_1, \ldots, t_{\frac{n}{2}}, f_1, \ldots, f_{\frac{n}{2}}) \in$ Seq$(\mathbb{R})$.

Higher-Level Haar Transforms

For $\mathbf{x} \in \mathbb{R}^n$ and $k = \log_2 n$:

$$
\begin{aligned}
\mathcal{H}^{[1]}(\mathbf{x}) &= \mathcal{H}(\mathbf{x}) = (\mathbf{t}^1|\mathbf{f}^1), \\
\mathcal{H}^{[2]}(\mathbf{x}) &= (\mathcal{H}(\mathbf{t}^1)|\mathbf{f}^1) = (\mathbf{t}^2|\mathbf{f}^2|\mathbf{f}^1), \\
\mathcal{H}^{[3]}(\mathbf{x}) &= (\mathcal{H}(\mathbf{t}^2)|\mathbf{f}^2|\mathbf{f}^1) = (\mathbf{t}^3|\mathbf{f}^3|\mathbf{f}^2|\mathbf{f}^1), \\
&\vdots \\
\mathcal{H}^{[k]}(\mathbf{x}) &= (\mathbf{t}^k|\mathbf{f}^k|\mathbf{f}^{k-1}|\cdots|\mathbf{f}^1)
\end{aligned}
$$

The Full Haar Transform

The *full Haar transform* of a sequence **x** of length n is

$$\mathbf{H}(\mathbf{x}) = (\mathbf{t}^k|\mathbf{f}^k|\mathbf{f}^{k-1}|\cdots|\mathbf{f}^1),$$

where $k = \log_2 n$.

Energy Localization Property

Most of energy is concentrated in the trend vector. For

$$\mathbf{x} = (2, 3, 5, 4, 2, 6, 8, 10)$$

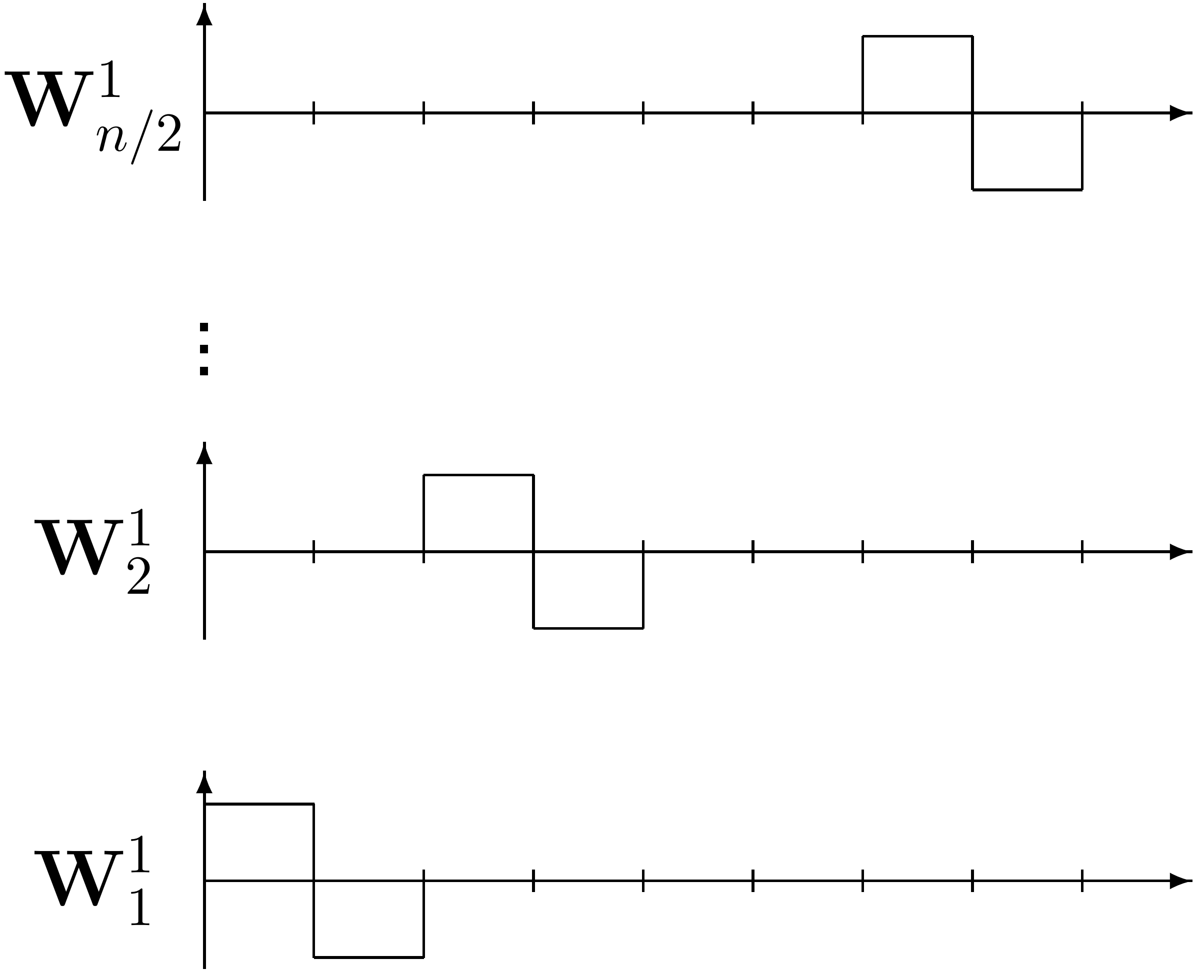
we have

$$\mathcal{E}(\mathbf{x}) = 2^2 + 3^2 + 5^2 + 4^2 + 2^2 + 6^2 + 8^2 + 10^2 = 258$$
$$\mathcal{E}(\mathbf{t}^1) = 3.54^2 + 6.36^2 + 5.65^2 + 12.85^2 \approx 246$$
$$\mathcal{E}(\mathbf{f}^1) = 0.70^2 + 0.70^2 + 2.80^2 + 1.40^2 \approx 12$$

Haar Wavelets

The *1-level Haar wavelets* are the sequences

$$
\begin{aligned}
\mathbf{W}^1_1 &= (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, \ldots, 0, 0) \\
\mathbf{W}^1_2 &= (0, 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \ldots, 0, 0) \\
&\vdots \\
\mathbf{W}^1_{\frac{n}{2}} &= (0, 0, 0, 0, \ldots, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})
\end{aligned}
$$


$\mathbf{W}^1_{n/2}$
⋮
$\mathbf{W}^1_2$
$\mathbf{W}^1_1$

A Bit of History ...

- Weierstrass (1873): a family of functions constructed by superimposing scaled copies of a given base function.
- Haar (1909): introduced the Haar basis (compact support).
- Gabor (1946): nonorthogonal basis of functions with unbounded support (translations of Gaussians).
- Ricker (1940): the term wavelet (seismology)

Properties of Wavelets

If $\mathcal{H}(\mathbf{x}) = (\mathbf{t}|f_1, \ldots, f_{\frac{n}{2}})$, then

$$f_i = \mathbf{x}\mathbf{W}_i \text{ for } 1 \leq i \leq \frac{n}{2}.$$

- Average value of a wavelet is 0.
- For each wavelet $\mathbf{W}_i^1$ we have $\mathcal{E}(\mathbf{W}_i^1) = 1$.
- Each wavelet can be obtained from the first wavelet by a time-translation of 2.
- If $\mathbf{x}$ is approximatively constant on $\mathrm{supp}(W_i^1)$, then f_i is approximatively 0.

The Haar Scaling signals

The *1-level Haar scaling signals* are:

$$
\begin{aligned}
\mathbf{V}_1^1 &= (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, \ldots, 0, 0) \\
\mathbf{V}_2^1 &= (0, 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \ldots, 0, 0) \\
&\vdots \\
\mathbf{V}_{\frac{n}{2}}^1 &= (0, 0, 0, 0, \ldots, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})
\end{aligned}
$$

Properties of Scaling Signals

If $\mathcal{H}(\mathbf{x}) = (t_1, \ldots, t_{\frac{n}{2}} | \mathbf{f})$, then

$$t_i = \mathbf{x}\mathbf{V}_i^1 \text{ for } 1 \leq i \leq \frac{n}{2}.$$

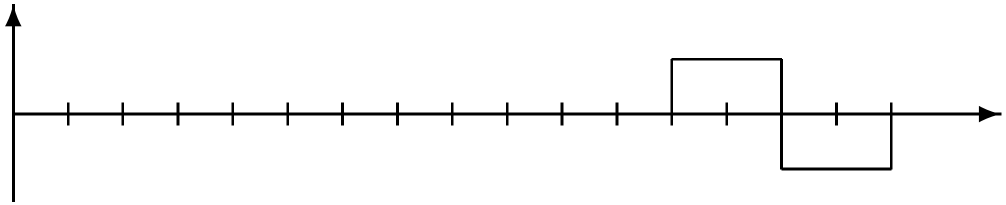
- Average value of a scaling signal is not 0.
- For each scaling signal $\mathbf{V}_i^1$ we have $\mathcal{E}(\mathbf{V}_i^1) = 1$.
- Each scaling signal can be obtained from the first scaling signal by a time-translation of 2.

2nd-Level Wavelets

The *2nd-level wavelets* are defined by

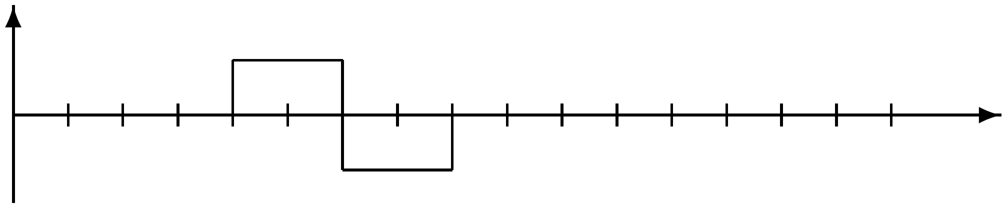
$$\begin{aligned}
\mathbf{W}_1^2 &= (\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 0, \ldots, 0) \\
\mathbf{W}_2^2 &= (0, 0, 0, 0\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 0, \ldots, 0) \\
&\vdots \\
\mathbf{W}_{\frac{n}{4}}^2 &= (0, 0, 0, 0, 0, \ldots, 0, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})
\end{aligned}$$

$\mathbf{W}^2_{n/4}$

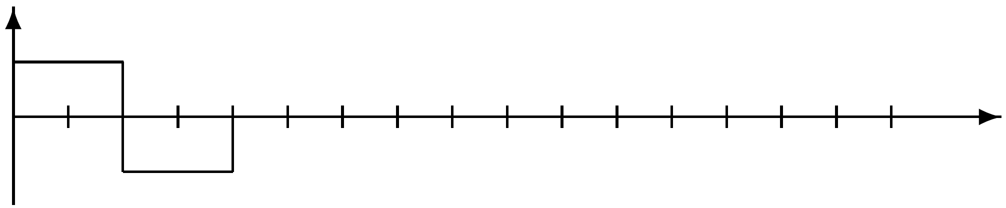


$\vdots$

$\mathbf{W}^2_2$



$\mathbf{W}^2_1$



2nd-Level Scaling Signals

The *2nd-level scaling* are defined by

$$
\begin{aligned}
\mathbf{V}_1^2 &= (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, \ldots, 0) \\
\mathbf{V}_2^2 &= (0, 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, \ldots, 0) \\
&\vdots \\
\mathbf{V}_{\frac{n}{4}}^2 &= (0, 0, 0, 0, 0, \ldots, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})
\end{aligned}
$$

2nd-order fluctuations:

$$f_i^2 = \textbf{xW}_i^2 \text{ for } 1 \leq i \leq \frac{n}{4}$$

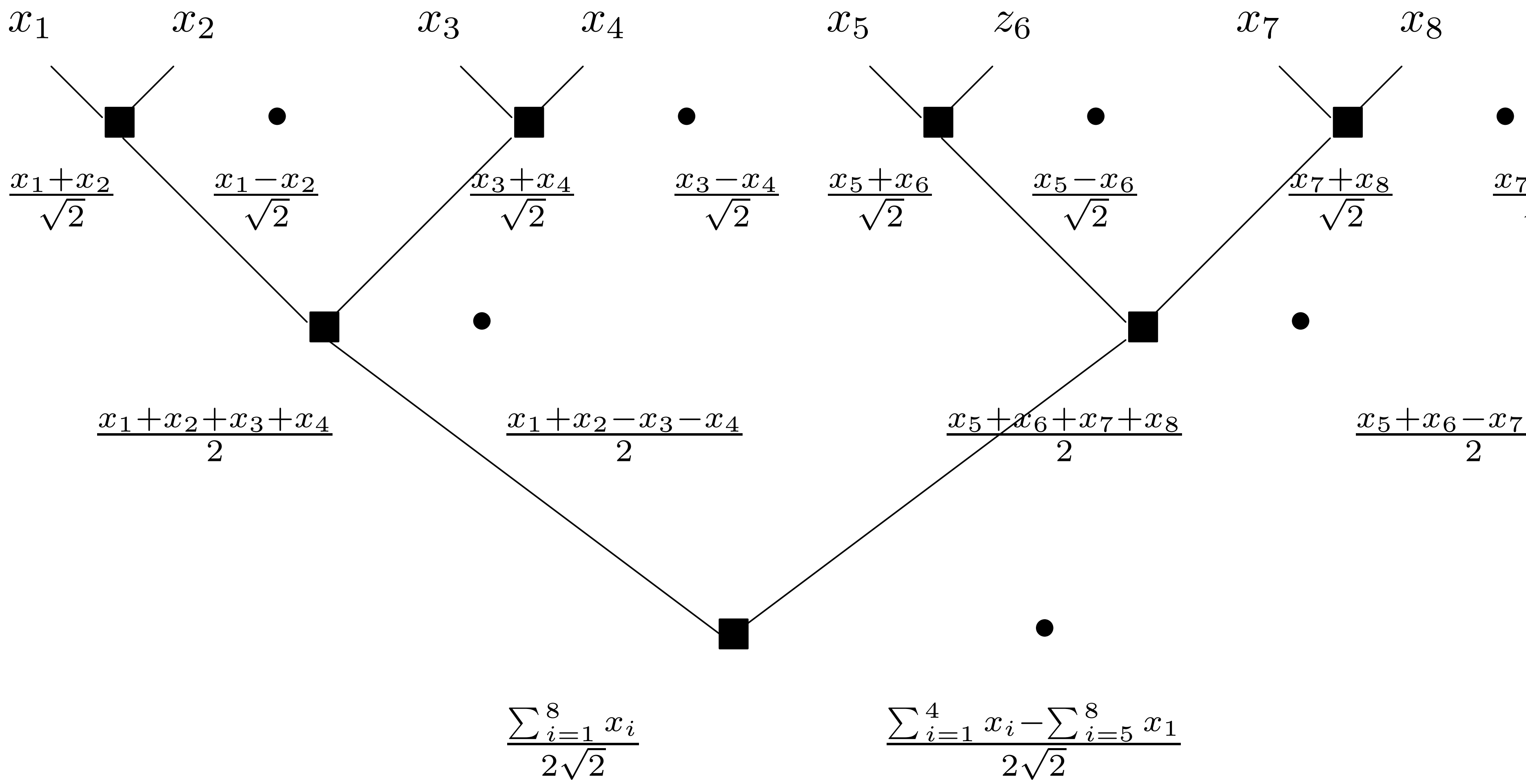
2nd-order trends:

$$t_i^2 = \textbf{xV}_i^2 \text{ for } 1 \leq i \leq \frac{n}{4}$$

Properties of the 2nd-level wavelets anc

- Average value of wavelets is 0; average value of scaling signals is non-zero.
- $\mathcal{E}(\mathbf{W}_i^2) = \mathcal{E}(\mathbf{V}_i^2) = 1$,
- $\text{supp}(\mathbf{W}_i^2) = \text{supp}(\mathbf{V}_i^2) = 4,$ for $1 \leq i \leq \frac{n}{4}$.

Computation Tree



Full System of Wavelets

For $1 \leq j \leq \log_2 n$ and $1 \leq h \leq \frac{n}{2^j}$ define:

$$\mathbf{W}_h^j = (0, \ldots, 0, \underbrace{\left(\frac{1}{\sqrt{2}}\right)^j, \ldots, \left(\frac{1}{\sqrt{2}}\right)^j}_{2^{j-1}}, \underbrace{-\left(\frac{1}{\sqrt{2}}\right)^j, \ldots, -\left(\frac{1}{\sqrt{2}}\right)^j}_{2^{j-1}}, 0, \ldots, 0)$$

Full System of Wavelets for $n = 8$

$$W_1^1 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0\right) \qquad W_2^1 = \left(0, 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0\right)$$

$$W_3^1 = \left(0, 0, 0, 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0\right) \qquad W_4^1 = \left(0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

$$W_1^2 = \left(\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, 0, 0, 0, 0\right) \qquad W_2^2 = \left(0, 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}\right)$$

$$W_1^3 = \left(\left(\frac{1}{\sqrt{2}}\right)^3, \left(\frac{1}{\sqrt{2}}\right)^3, \left(\frac{1}{\sqrt{2}}\right)^3, \left(\frac{1}{\sqrt{2}}\right)^3, -\left(\frac{1}{\sqrt{2}}\right)^3, -\left(\frac{1}{\sqrt{2}}\right)^3, -\left(\frac{1}{\sqrt{2}}\right)^3, -\left(\frac{1}{\sqrt{2}}\right)^3\right)$$

Full System of Wavelets for $n = 8$

Full Haar transform of a sequence **x**:

$$(\mathbf{x}\mathbf{V}_1^3, \mathbf{x}W_1^3, \mathbf{x}W_1^2, \mathbf{x}W_2^2,$$
$$\mathbf{x}W_1^1, \mathbf{x}W_2^1, \mathbf{x}W_3^1, \mathbf{x}W_4^1)$$

Example: For $\mathbf{x} = (2, 3, 5, 4, 2, 6, 8, 10)$:

$$\begin{aligned} \mathbf{H}(\mathbf{x}) &= (14.2, -4.30, -1.99, -5.09, \\ &\quad -0.70, 0.70, -2.80, -1.40) \end{aligned}$$

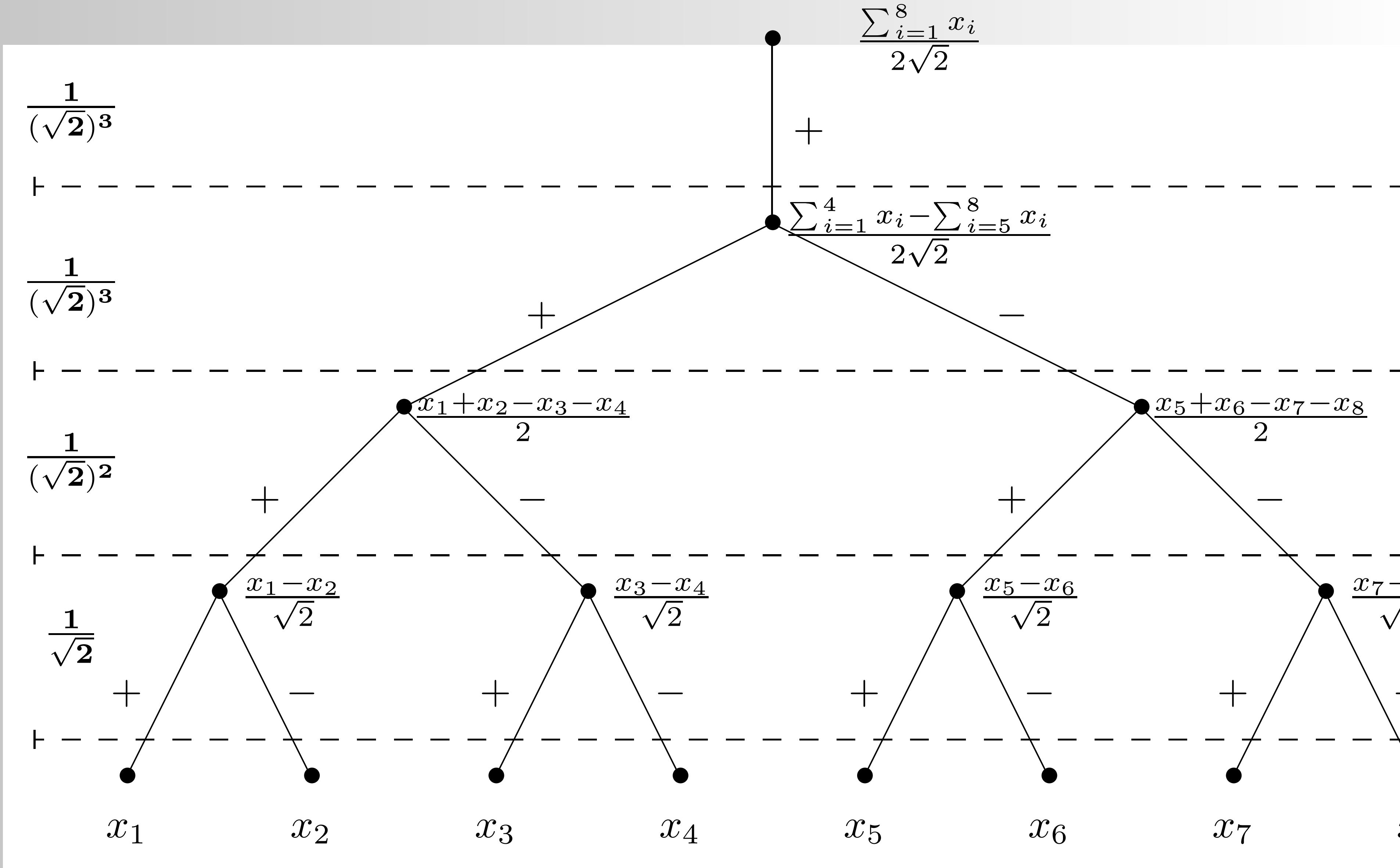
Synthesis of Signals

The inverse Haar transform:

$$x_1 = \frac{t_1 + f_1}{\sqrt{2}}, x_2 = \frac{t_1 - f_1}{\sqrt{2}}, \ldots, x_n = \frac{t_{\frac{n}{2}} - f_{\frac{n}{2}}}{\sqrt{2}}$$

$$\mathbf{x} = \left(\frac{t_1}{\sqrt{2}}, \frac{t_1}{\sqrt{2}}, \ldots, \frac{t_{\frac{n}{2}}}{\sqrt{2}}, \frac{t_{\frac{n}{2}}}{\sqrt{2}}\right) + \left(\frac{f_1}{\sqrt{2}}, -\frac{f_1}{\sqrt{2}}, \ldots, \frac{f_{\frac{n}{2}}}{\sqrt{2}}, -\frac{f_{\frac{n}{2}}}{\sqrt{2}}\right)$$

The Inverse of the full Haar Transform



Multiresolution Analysis

First averaged and first detail signals are:

$$\mathbf{T}^1 = \left(\frac{t_1}{\sqrt{2}}, \frac{t_1}{\sqrt{2}}, \ldots, \frac{t_{\frac{n}{2}}}{\sqrt{2}}, \frac{t_{\frac{n}{2}}}{\sqrt{2}}\right)$$

$$\mathbf{F}^1 = \left(\frac{f_1}{\sqrt{2}}, -\frac{f_1}{\sqrt{2}}, \ldots, \frac{f_{\frac{n}{2}}}{\sqrt{2}}, -\frac{f_{\frac{n}{2}}}{\sqrt{2}}\right)$$

$\mathbf{x} = \mathbf{T}^1 + \mathbf{F}^1$: sum of a lower resolution signal and a detail signal.

Averaged and Detail Signals (cont)

The averaged and detail signals can be written as

$$\begin{aligned} \mathbf{T}^1 &= t_1 \mathbf{V}_1^1 + \cdots + t_{\frac{n}{2}} \mathbf{V}_{\frac{n}{2}}^1 \\ &= (\mathbf{x}\mathbf{V}_1^1)\mathbf{V}_1^1 + \cdots + (\mathbf{x}\mathbf{V}_{\frac{n}{2}}^1)\mathbf{V}_{\frac{n}{2}}^1 \\ \mathbf{F}^1 &= f_1 \mathbf{W}_1^1 + \cdots + f_{\frac{n}{2}} \mathbf{W}_{\frac{n}{2}}^1 \\ &= (\mathbf{x}\mathbf{W}_1^1)\mathbf{W}_1^1 + \cdots + (\mathbf{x}\mathbf{W}_{\frac{n}{2}}^1)\mathbf{W}_{\frac{n}{2}}^1 . \end{aligned}$$

Example:

Let $\mathbf{x} = (2,3,5,4,2,6,8,10)$. We have

$$\begin{aligned}\mathcal{H}(\mathbf{x}) \;&=\; (\frac{5}{\sqrt{2}}, \frac{9}{\sqrt{2}}, \frac{8}{\sqrt{2}}, \frac{18}{\sqrt{2}}| \\ &\quad -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{4}{\sqrt{2}}, -\frac{2}{\sqrt{2}})\end{aligned}$$

The averaged signal:
$\mathbf{T}^1 = (5/2, 5/2, 9/2, 9/2, 4, 4, 9, 9)$
The detail signal:

$\mathbf{F}^1 = (-1/2, 1/2, 1/2, -1/2, -2, 2, -1, 1)$

Multiple-level MRA

$$\begin{aligned} \mathbf{x} &= \mathbf{T}^1 + \mathbf{F}^1 \\ \mathbf{T}^1 &= \mathbf{T}^2 + \mathbf{F}^2 \\ &\vdots \\ \mathbf{T}^{k-1} &= \mathbf{T}^k + \mathbf{F}^k, \end{aligned}$$

so

$$\mathbf{x} = \mathbf{T}^k + \mathbf{F}^k + \mathbf{F}^{k-1} + \cdots + \mathbf{F}^1$$

Compression of Signals

- Compression: converting a signal into a new format that requires fewer bits to transmit
- Categories of compression
  - *lossless copmpression*: error-free decompression of the original signal (Huffman compression, LZW compression, arithmetic compression)
  - *lossy compression*: produces inaccuracies in the decompressed signal
- rates of compression (50:1–100:1) for lossy compr. vs. 2:1 for lossless

Wavelet Compression Methods

1. Compute a wavelet transform of a signal.
2. Set to 0 all values of components that are below a threshold value λ.
3. Transmit only the significant, non-zero values.
4. Compute the reverse transform at the receiving end, using zero values forthe components that were not transmitted.

A Relational Database Application

Selectivity Estimation

ORDERS

cust_no	cust_name	date	qty
123	John Doe	2/10/2003	8
⋮	⋮	⋮	⋮

Find the fraction of ORDERS returned by:

select cust_name **from** ORDERS
where 1 <= qty **and** qty <= 3;

Wavelet-based Histograms (Vitter)

The active domain of A: $v_1 < v_2 < \cdots < v_n$: the values that appear under an attribute A of a table.

Frequencies: $f_i = |\{t|t[A] = v_i\}|$, $1 \leq i =\leq n - 1$

Cumulative Frequencies:

$$c_i = |\{t|t[A] <= v_i\}| = \sum_{k=1}^{i} f_k,$$

for $1 \leq i =\leq n - 1$

Cumulative Data Distribution

Data distribution of A:

$$\mathcal{T}(A) = \{(v_1, f_1), \ldots, (v_n, f_n)\}$$

Cumulative data distribution of A:

$$\mathcal{T}^C(A) = \{(v_1, f_1), \ldots, (v_n, f_n)\}$$

Extended cumulative data distribution $\mathcal{T}^{C+}(A)$ is the extension of $\mathcal{T}^C$ obtained by assigning 0 frequencies to all values that do not occur in the table.

Vitters' Histogram Construction

1. form the extended cummulative distribution $\mathcal{T}^{C+}(A)$ (preprocessing);
2. compute $\mathcal{H}(\mathcal{T}^{C+}(A))$;
3. retain only the m most significant wavelet coefficients for some m that corresponds to the desired storage usage.

The number of tuples $T(A)_{a,b}$ such that $a \leq A \leq b$ is

$$T(A)_{a,b} = \mathcal{T}^{C+}(A)_b - \mathcal{T}^{C+}(A)_{a-1}$$

Example:

ORDERS	
...	qty
	1
	3
	4
	3
	1
	4
	3
	3
	3

$$\mathcal{T}(\text{qty}) = \{(1,2),(3,5),(4,2)\}$$

$$\mathcal{T}^{C+}(\text{qty}) = \{(1,2),(2,2),(3,7),(4,9)\}$$

$$\mathcal{H}(\mathcal{T}^{C+}(2,2,7,9)) = (9.99,-5.99,0,-1.91)$$

$$\mathcal{H}^{-1}(\mathcal{T}^{C+}(9.99,-5.99,0,-1.91)) = (1.99,1.99,6.99,8.99)$$

Further steps and remarks ...

- The value of the m coefficients together with their positions serve as histogram.
- To estimate the value of $|\{t|c \geq t[A] \geq d\}|$ we construct the values for b and $a-1$ in in the extended cumulative distribution function and then take their difference.
- Effectiveness is increased when we replace the raw frequencies with $\mathcal{T}^{C+}(A)$.

Preprocessing

- If the active domain V is small, an one-pass, in-memory computation is sufficient.
- If V is large, use an external merge-sort and sum up the frequencies of the records that are merged.
- If V is very large use random sampling and use the sample data distribution as an approximation.

Restricting the Coefficients

Thresholding: m out of N coefficients are kept; the remaining are set to 0. Then, the inverse Haar transform is computed.
Let s be the size of query q and s' be the size of query q after thresholding.
Error computations for a query q_i:

- absolute error: $e^{abs}(q) = |s - s'|$ (small freqs.)
- relative error: $e^{rel}(q) = \frac{e^{abs}}{s}$ (large freqs.)
- combined error: $e^{comb}(q) = \min\{\alpha e^{abs}(q), \beta e^{rel}(q)\}$

Global error for a set of queries

For a set $Q = \{q_1, \ldots, q_k\}$ of queries we have an error vector

$$\mathbf{e} = (e(q_1), \ldots, e(q_k))$$

The overall error is

$$||e||_p = \left(\frac{1}{k} \sum_{i=1}^{k} e_i^p \right)^{\frac{1}{p}}$$

Thresholding Techniques

- Choose the largest m wavelet coefficients in absolute value.
- Choose m coefficients in a greedy way (e.g. as above), then repeatedly include the coefficients that decrease the error and exclude those that increase it.

Estimating Selectivity

Vitter's Theorem: For a given range query $a \leq X \leq b$, the cumulative frequencies of $a-1$ and b can be reconstructed from m wavelet coefficients using $O(m)$ space in time $O(\min\{m, \log N\})$.

Mining Data Streams

Mining data that arrives and is processed in a stream: *"you look only once"*
Examples:

- switches and routers in networks generate data on
  - telephone calls
  - IP addresses
- streams of credit card transactions
- log records in web-based services

Main Challenge:

Data accumulation is expensive so it is important to extract information even at the cost of obtaining approximative results.

The Processing Model

Characteristics of stream processing are identified:

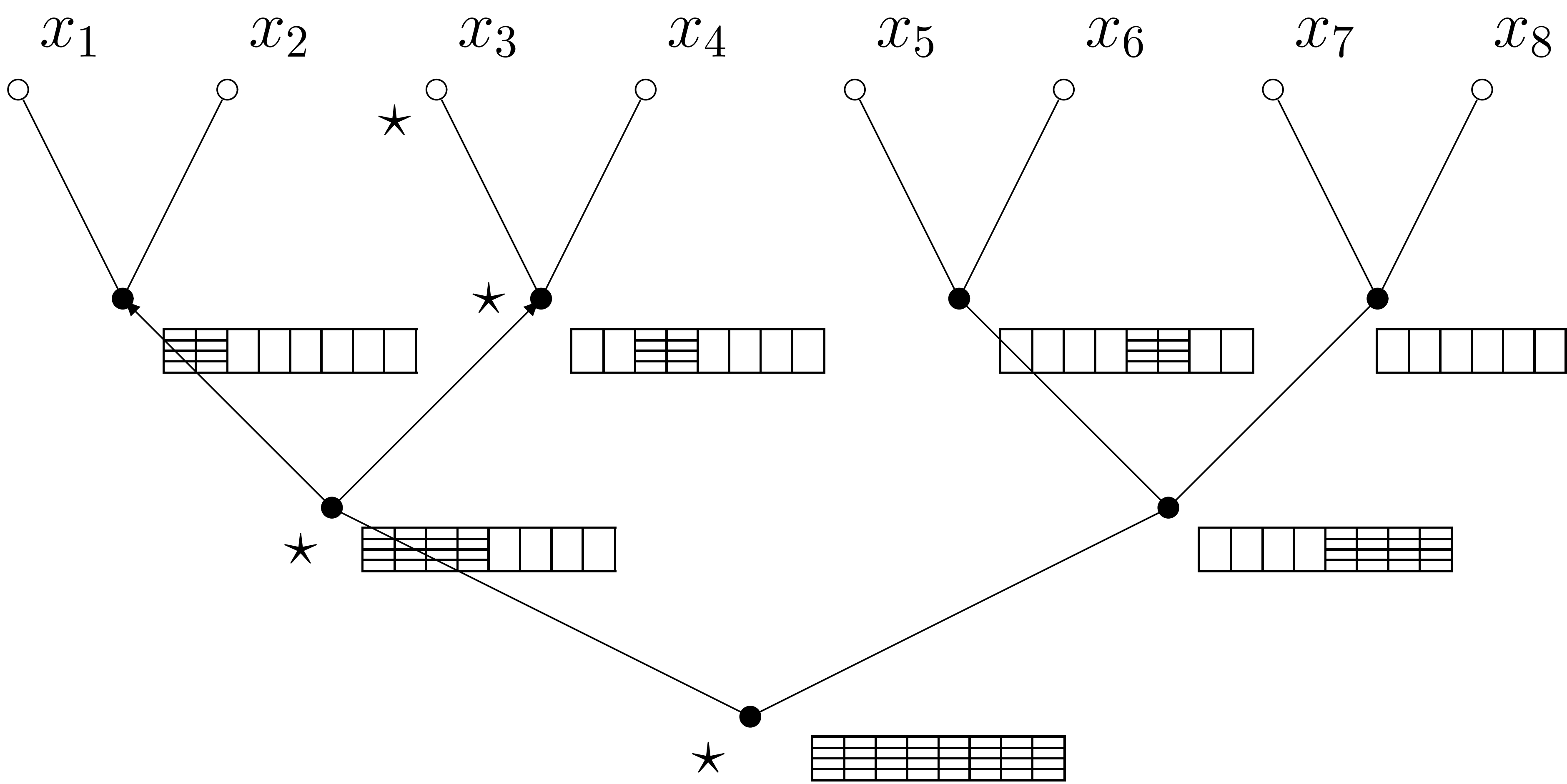
- each data item is read and processed as soon as it arrives;
- no backtracking is allowed on the data stream;
- explicit access to arbitrary past items is not allowed.

What is allowed ...

An additional amount of memory is permitted subjected to the following conditions:

- the additional memory may be used to store:
  - a recent window of items;
  - some sumary information about the stream.
- the size of the memory is significantly smaller than the signal domain size.

Straddling Coeficients



Computation of the higest m terms

The highest m terms yields the best approximation for the error $||e||_2$.

Gilbert's result: With the most $O(m + \log N)$ storage we can compute the highest m-term approximation to a signal. Each new data signal item needs $O(m + \log n)$ time to be processed.

Lower Space Bound

Any streaming algorithm that correctly calculates the highest wavelet basis coefficient of a signal requires $\Omega(\frac{N}{\log \log N})$ space.

Other Applications

- Clustering time series that represent levels of gene expressions in microarrays as they appear in the mitosis process (a study of cellular division of the cells that form the retina).
- The new image data compression standard JPEG 2000

Conclusions

- Wavelet transforms generate simple algorithms for data compression.
- Computations can be done efficiently, in small space.
- A large variety of applications exist even for the simplest Haar wavelets.