

Fingerprint-based Location Tracking with Hodrick-Prescott Filtering

Duc A. Tran Ting Zhang
Department of Computer Science
University of Massachusetts, Boston, MA
Email: {duc.tran, ting.zhang001}@umb.edu

Abstract—Conventional location fingerprint techniques usually require a prebuilt training set of fingerprints sampled at known locations, so that locations of future fingerprints can be determined by comparing to this set. For good accuracy, the training set should be large enough to appropriately cover the area. However, it is not always feasible to obtain a quality training set in practice, and so recent studies have resorted to utilizing fingerprints that are available but without location information. This paper investigates how these so-called unlabeled fingerprints can be useful for location tracking of a mobile device as it is moving. Specifically, we propose a fingerprint-based tracking approach based on Hodrick-Prescott filtering and substantiate its potential via an evaluation study.

I. INTRODUCTION

Location information is valuable to a myriad of applications of wireless networks. In a surveillance sensor network, it is crucial to know the location of an incident caught by a sensor, such as fire in a building or oil spill in a coastal water. The demand is also high for mobile apps providing navigation and other location-based services in hospitals, shopping malls, airport terminals, and campus buildings, to name a few. GPS is the most effective way to get location information but does not work indoors. Even for outdoor environments where this service is available, it is not energy-efficient to have to turn it on continuously all the time.

Consequently, numerous efforts have been made towards GPS-free localization solutions, most adopting the fingerprint approach. This approach usually consists of two phases: training (offline) and positioning (online). In the offline phase, a number of sample locations are surveyed to build a map corresponding each location to a “fingerprint” which is a vector of measurements observed between the mobile device at this location and a set of “reference points” (RPs). For example, these RPs can be a set of Wi-Fi access points [1], FM broadcasting towers [2], or cellular towers [3], and measurements can be the received signal strength indices (RSSI) observed between them and the mobile device. In the online phase, when we need to compute a location in real time, the current fingerprint of the device is compared against the fingerprint map to find the best location match. Recently, fingerprint modalities other than radio have been suggested, such as sound [4] and geomagnetic field [5]. By combining these different features where they apply, we can obtain a rich set of discriminative features for the fingerprint information.

The localization accuracy of the fingerprint approach largely depends on the quality of the training data. Due to the tedium and labor cost of the calibration task in the offline phase, it is not always feasible to obtain a quality fingerprint map. In such a case, a viable approach [6]–[10] for better accuracy is to apply a semi-supervised learning method taking into account the availability of “unlabeled” fingerprints (whose location is unknown), not just the set of “labeled” fingerprints (whose location is known). Unlabeled fingerprints are abundant and can easily be obtained without manual location labeling.

This paper is focused on the problem of tracking a mobile device based on its sequentially obtained fingerprints. We do not require a prebuilt map of labeled fingerprints. Instead, we only assume that once in a while a fingerprint is observed with a known location. This assumption is necessary for otherwise it is impossible to make any inference about the device’s location. It is noted that while there have been earlier research works on mobile location tracking, most of them make additional assumptions, such that those about special sensors built in the device (e.g., gyroscope, accelerometer, compass, camera) [11], [12], those about mobility-specific constraints (e.g., speed, predefined map) [13] and those that are network-specific (e.g., vehicular or wireless sensor networks) [14], [15]. In contrast, we are interested in fingerprint-based tracking and aim to devise a tracking framework with universal applicability in the sense that it can work orthogonally with any type of fingerprint space; i.e., applicable where fingerprint information can be of radio signals, acoustic, or geomagnetic, etc and can contain any other information that is location-discriminative.

Our intuition is that as the device is moving its fingerprints should satisfy two properties: spatial smoothness and temporal smoothness. Fingerprints having similar values, regardless of their observation time, should correspond to nearby locations (spatial smoothness) and fingerprints observed in consecutive movements of the device should also correspond to nearby locations resulted from a constant speed (temporal smoothness). The spatial smoothness property has been exploited in earlier location fingerprint techniques [6]–[10]. They commonly formulate the localization problem as a manifold regularization problem [16] which includes a regularizer term to maximize spatial smoothness. In this paper, we want to investigate how useful the temporal smoothness property can be in order to improve localization accuracy. Specifically, we formulate the

fingerprint-based tracking problem as a regularization problem extended with a Hodrick-Prescott (HP) filter [17] term to regulate the temporal smoothness. The location estimation algorithm, as a solution to this problem, is faster than the manifold regularization based algorithm, and as shown in our evaluation study, more accurate.

The remainder of the paper is structured as follows. §II provides a brief survey of the related work. §III presents the details of our proposed approach to the fingerprint-based tracking problem. Evaluation results are discussed in §IV. The paper is concluded in §V with pointers to our future work.

II. RELATED WORK

GPS-free localization in wireless networks has been a long-studied problem. There exist many techniques to date, which differ in the type of network environment (e.g., sensor networks [18], wireless LANs [19], vehicular ad hoc networks [14]), the modality of information used to infer location (e.g., infrared [20], radio [1], sound [4], geo-magnetic [5], light [21]), or the type of algorithmic method (e.g., range-based [22], range-free [23]).

Radar [1] is the world's first Wi-Fi RSS-based indoor positioning system, which demonstrates the viability of using RSS information to locate a wireless device. This system works using a radio map, a lookup table that maps building locations to the corresponding RSS fingerprints empirically observed at these locations. The reference points are the Wi-Fi access points within the user's Wi-Fi range. The radio map is searched to find the closest RSS readings and the centroid of the corresponding locations will be used as the estimate for the user's location. Radar represents the fingerprint approach where kNN is used to determine the location. One can also employ other learning methods to relate a fingerprint to a location, such as probabilistically using Bayesian inference [24] or non-probabilistically using an Artificial Neural Network (ANN) [25] or a Support Vector Machine (SVM) [26].

When there are only a small number of sample fingerprints for training, we can utilize unlabeled fingerprints as a supplement to the original ones by solving a manifold regularization problem, a widely-used semi-supervised learning method of by Belkin et al. in [16] in the area of Machine Learning, to propagate the labels for the unlabeled fingerprints based on their similarity with the labeled. Pan et al. [6], [7] apply manifold regularization with a Laplacian regularization term reflecting the intrinsic manifold structure of the fingerprints; here the manifold is a weighted graph of fingerprints in which the weight of an edge connecting two fingerprints represents the similarity between them. Other regularization terms have also been investigated. For example, e.g., Total Variation [27] considered in the recent work of Tran and Truong [10]; this work, however, suggests that manifold regularization with the Laplacian term offers better localization accuracy than TV.

Our research in this paper also applies a regularization framework for learning the location labels for the unlabeled fingerprints, but our regularization term is based on the Hodrick-Prescott filter [17]. Our focus of attention is on the

effectiveness of temporal smoothness in the fingerprint space for the location estimation, whereas the conventional manifold regularization approach focuses only on spatial smoothness. HP is an effective tool for trend estimates in time series. It has been used in the work of Rallapalli et al. [28] for mobile tracking, which solves an optimization problem with constraints and assumptions about device-to-device distance. In contrast, ours is the first effort to explore HP for fingerprint-based location tracking.

III. FINGERPRINT-BASED LOCATION TRACKING

Suppose that we need to compute the instant location of a given fingerprint that is obtained in a stream manner, $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_t, \dots$, where the time is discretized into time steps 1, 2, ..., t , ... Each fingerprint is a m -dimensional point, $\mathbf{x}_t \in \mathcal{X} \subset \mathbb{R}^m$, where m is the number of fingerprint features, e.g., RSSI from different Wi-Fi APs, readings from inertial measurement units (accelerometer, gyroscope, magnetometer), and any location-discriminative feature that is available with the device, etc. Denote by $\mathbf{y}_t \in \mathbb{R}^d$ the location corresponding to \mathbf{x}_t , where d is the dimensionality of the location space. For ease of presentation, let $d = 1$ and so $\mathbf{y}_t = y_t$ is a real-valued number; we will discuss the case $d > 1$ later.

We use the notation h_t to represent whether a fingerprint x_t is labeled with location ($h_t = 1$) or unlabeled ($h_t = 0$). The labeled fingerprints become available only once in a while, one at a time but totally unpredictable. We want to find a real-time location estimator $f : \mathcal{X} \rightarrow \mathbb{R}$ that, upon receipt of a new fingerprint \mathbf{x}_t at the current time t , needs to output its corresponding location $f(\mathbf{x}_t)$. We formulate this problem as a regularization problem which utilizes information about both labeled and unlabeled fingerprints that have been obtained by the current time. In what follows, we present two approaches to this formulation. The first approach is the conventional formulation based on manifold regularization. The latter is the proposed formulation using Hodrick-Prescott filtering.

A. Manifold Regularization

Ideally, the location estimator f if applied on a labeled fingerprint should result in an estimate that matches its given location. Therefore, in search of f , a reasonable goal is to minimize the estimation error with respect to the labeled fingerprints. This is quantified by minimizing

$$\min_f \left\{ E(f) = \frac{1}{t} \sum_{i=1}^t h_i (f(\mathbf{x}_i) - y_i)^2 \right\}. \quad (1)$$

Another goal is to maximize the spatial smoothness in the fingerprint space. As aforementioned, similar fingerprints, regardless of when they are observed, should correspond to nearby locations. We quantify this by, first, organizing the fingerprints into an undirected weighted graph, where each vertex is a fingerprint and each edge has a weight $w(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{|\mathbf{x}_i - \mathbf{x}_j|^2}{2\sigma^2}\right)$ (for some constant σ) reflecting the similarity between \mathbf{x}_i and \mathbf{x}_j , and, second, minimizing the

Laplacian quadratic form of this graph:

$$\min_f \left\{ S(f) = \frac{1}{t^2} \sum_{i=1}^t \sum_{j=1}^i w(\mathbf{x}_i, \mathbf{x}_j) (f(\mathbf{x}_i) - f(\mathbf{x}_j))^2 \right\}. \quad (2)$$

To optimize these two goals, we combine them into a single objective function using Belkin et al.'s manifold regularization framework [16]. Specifically, the location estimator f is the solution minimizing the following risk:

$$\min_f \{J(f) = E(f) + \lambda_S S(f)\}, \quad (3)$$

where coefficient $\lambda_S > 0$ reflects the importance of spatial smoothness maximization.

The beauty of this approach is that we can easily derive a closed form for the location estimator f . Denote the following vectors and matrices: $\mathbf{f} = [f(\mathbf{x}_1), f(\mathbf{x}_2), \dots, f(\mathbf{x}_t)]^\top$, $\mathbf{y} = [y_1, y_2, \dots, y_t]^\top$ (y_i is set to zero by default for unlabeled \mathbf{x}_i), $\mathbf{H} = \text{diag}(h_1, h_2, \dots, h_t)$, the identity matrix $\mathbf{I} = \text{diag}(\underbrace{1, 1, \dots, 1}_t)$, and the Laplacian matrix \mathbf{L} of the weighted fingerprint graph. Then we can express the functionals in Eqs. (1) and (2) in matrix form as follows, $E(f) = \frac{1}{t}(\mathbf{f} - \mathbf{y})^\top \mathbf{H}(\mathbf{f} - \mathbf{y})$ and $S(f) = \frac{1}{t^2} \mathbf{f}^\top \mathbf{L} \mathbf{f}$. Thus, the risk $J(f)$ in Eq. (3) can be expressed in matrix form as

$$\begin{aligned} \mathbf{J} &= \frac{1}{t}(\mathbf{f} - \mathbf{y})^\top \mathbf{H}(\mathbf{f} - \mathbf{y}) + \lambda_S \frac{1}{t^2} \mathbf{f}^\top \mathbf{L} \mathbf{f} \\ &= \frac{1}{t} \left(\underbrace{\mathbf{f}^\top (\mathbf{H} + \frac{\lambda_S}{t} \mathbf{L}) \mathbf{f}}_{\mathbf{Q}} - 2\mathbf{y}^\top \mathbf{H} \mathbf{f} + \mathbf{y}^\top \mathbf{H} \mathbf{y} \right) \end{aligned}$$

To minimize \mathbf{J} , set its derivative with respect to \mathbf{f} to zero,

$$\frac{\partial \mathbf{J}}{\partial \mathbf{f}} = \frac{1}{t} [(\mathbf{Q} + \mathbf{Q}^\top) \mathbf{f} - 2\mathbf{H} \mathbf{y}] = \mathbf{0}. \quad (4)$$

Because of the symmetry of matrices \mathbf{H} and \mathbf{L} , we have $\mathbf{Q} + \mathbf{Q}^\top = 2\mathbf{Q}$ and so Eq. (4) leads to

$$\mathbf{f} = \underbrace{\left(\mathbf{H} + \frac{\lambda_S}{t} \mathbf{L} \right)^{-1}}_{\mathbf{f}_{MR}} \mathbf{H} \mathbf{y}. \quad (5)$$

With this location estimator, the location estimate for fingerprint \mathbf{x}_t will be the corresponding element (last element) in vector \mathbf{f} .

B. Hodrick-Prescott Filtering

The Hodrick-Prescott (HP) filter [17] is a mathematical tool used to obtain a smoothed-curve representation of a time series, one that is more sensitive to long-term than to short-term fluctuations. We propose to apply HP to our location tracking problem because the trajectory of a moving device should be smooth over time and should exhibit a trend; real-world mobility often exhibits moving at a constant velocity for a long period of time before changing speed [28]. Treating the sequence of fingerprints as a time series, and if the locations

of all these fingerprints are known, HP can be used to obtain a smoothed trajectory of locations, by solving the following problem:

$$\min_f \sum_{i=1}^t (f(\mathbf{x}_i) - y_i)^2 + \lambda_T \sum_{i=3}^t (f(\mathbf{x}_i) + f(\mathbf{x}_{i-2}) - 2f(\mathbf{x}_{i-1}))^2.$$

Here, the second term is added to penalize variations in the growth rate of the trend component. The argument appearing in the second term, $f(\mathbf{x}_i) + f(\mathbf{x}_{i-2}) - 2f(\mathbf{x}_{i-1})$, is the second difference of the time series at time t ; it is zero when and only when the points $f(\mathbf{x}_i)$, $f(\mathbf{x}_{i-1})$, and $f(\mathbf{x}_{i-2})$ are on a line. The HP trend estimate, as the solution to this optimization problem, is, in matrix form,

$$\mathbf{f} = (\mathbf{I} + \lambda_T \mathbf{D} \mathbf{D}^\top)^{-1} \mathbf{y},$$

where \mathbf{D} is the second-order difference matrix

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & \dots & \dots & \dots & 0 \\ 1 & -2 & 1 & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & 1 & -2 & 1 & 0 \\ 0 & \dots & \dots & \dots & \dots & 0 & 1 & -2 & 1 \end{bmatrix}_{t \times t}$$

Coefficient $\lambda_T > 0$ controls the smoothness of the trend estimation. If $\lambda_T \rightarrow 0$, the trend converges to the original time series data. On the other hand, if $\lambda_T \rightarrow \infty$, the trend is the best straight line fit to the time series data.

HP assumes that every point in the time series is labeled, whereas we do not know the location of every fingerprint in the fingerprint sequence. To integrate HP, we revise the optimization problem to find the location estimator as follows:

$$\min_f \sum_{i=1}^t h_i (f(\mathbf{x}_i) - y_i)^2 + \lambda_T \sum_{i=3}^t (f(\mathbf{x}_i) + f(\mathbf{x}_{i-2}) - 2f(\mathbf{x}_{i-1}))^2$$

or

$$\min_f \{J(f) = E(f) + \lambda_T T(f)\}, \quad (6)$$

where

$$T(f) = \frac{1}{t} \sum_{i=3}^t (f(\mathbf{x}_i) + f(\mathbf{x}_{i-2}) - 2f(\mathbf{x}_{i-1}))^2. \quad (7)$$

In matrix form, we have $T(f) = \mathbf{f}^\top \mathbf{D} \mathbf{D}^\top \mathbf{f}$. Thus, the risk $J(f)$ in Eq. (6) is

$$\begin{aligned} \mathbf{J} &= (\mathbf{f} - \mathbf{y})^\top \mathbf{H}(\mathbf{f} - \mathbf{y}) + \lambda_T \mathbf{f}^\top \mathbf{D} \mathbf{D}^\top \mathbf{f} \\ &= \mathbf{f}^\top (\mathbf{H} + \lambda_T \mathbf{D} \mathbf{D}^\top) \mathbf{f} - 2\mathbf{y}^\top \mathbf{H} \mathbf{f} + \mathbf{y}^\top \mathbf{H} \mathbf{y} \\ \Rightarrow \frac{\partial \mathbf{J}}{\partial \mathbf{f}} &= 2(\mathbf{H} + \lambda_T \mathbf{D} \mathbf{D}^\top) \mathbf{f} - 2\mathbf{H} \mathbf{y}. \end{aligned}$$

Setting $\partial \mathbf{J} / \partial \mathbf{f} = \mathbf{0}$, we have our location estimator as

$$\mathbf{f} = \underbrace{(\mathbf{H} + \lambda_T \mathbf{D} \mathbf{D}^\top)^{-1}}_{\mathbf{f}_{HP}} \mathbf{H} \mathbf{y}. \quad (8)$$



Fig. 1. Floor plan (68m \times 63m) with 208 sample locations, each represented by a black dot. The hallways are marked with white color and stairs with green. Black and blue areas are not penetrable. Three trajectories of different patterns and path lengths are shown in red color.

We have so far assumed that the location is 1D. For 2D or 3D localization, we simply apply the same algorithm separately for each coordinate. The computation of \mathbf{f}_{MR} in Eq. 5 involves inverting matrices of $t \times t$. Its computational complexity, therefore, is $O(t^3)$. The HP-based approach is much faster though. Note that

$$\mathbf{D}\mathbf{D}^\top = \begin{bmatrix} 1 & -2 & 1 & 0 & \dots & \dots & \dots & \dots & 0 \\ -2 & 5 & -4 & 1 & 0 & \dots & \dots & \dots & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & -4 & 6 & -4 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & \dots & \dots & \dots & 0 & 1 & -4 & 5 & -2 \\ 0 & \dots & \dots & \dots & \dots & 0 & 1 & -2 & 1 \end{bmatrix}_{t \times t}$$

is a pentadiagonal matrix and so is the matrix $(\mathbf{H} + \lambda_T \mathbf{D}\mathbf{D}^\top)$.

The inverse of a pentadiagonal matrix can be computed in $O(t)$ time; hence, $O(t)$ time to compute \mathbf{f}_{HP} in Eq. 8. This is a clear advantage of the HP-based approach in comparison to the manifold regularization approach.

IV. EVALUATION

We discuss the evaluation results in this section. We compare the HP filtering based approach to the manifold regularization approach, which for ease of presentation are referred to as HP and MR, respectively. As HP is obviously better than MR in terms of computation time, we focus on their location error as metric for comparison. This error (up to time t) is computed as the average “individual” location error for unlabeled fingerprints over the path traveled from the beginning (up to time t). The “individual” error corresponding to a fingerprint \mathbf{x}_t is the Euclidean distance between its location estimate at time t and its ground-truth location.

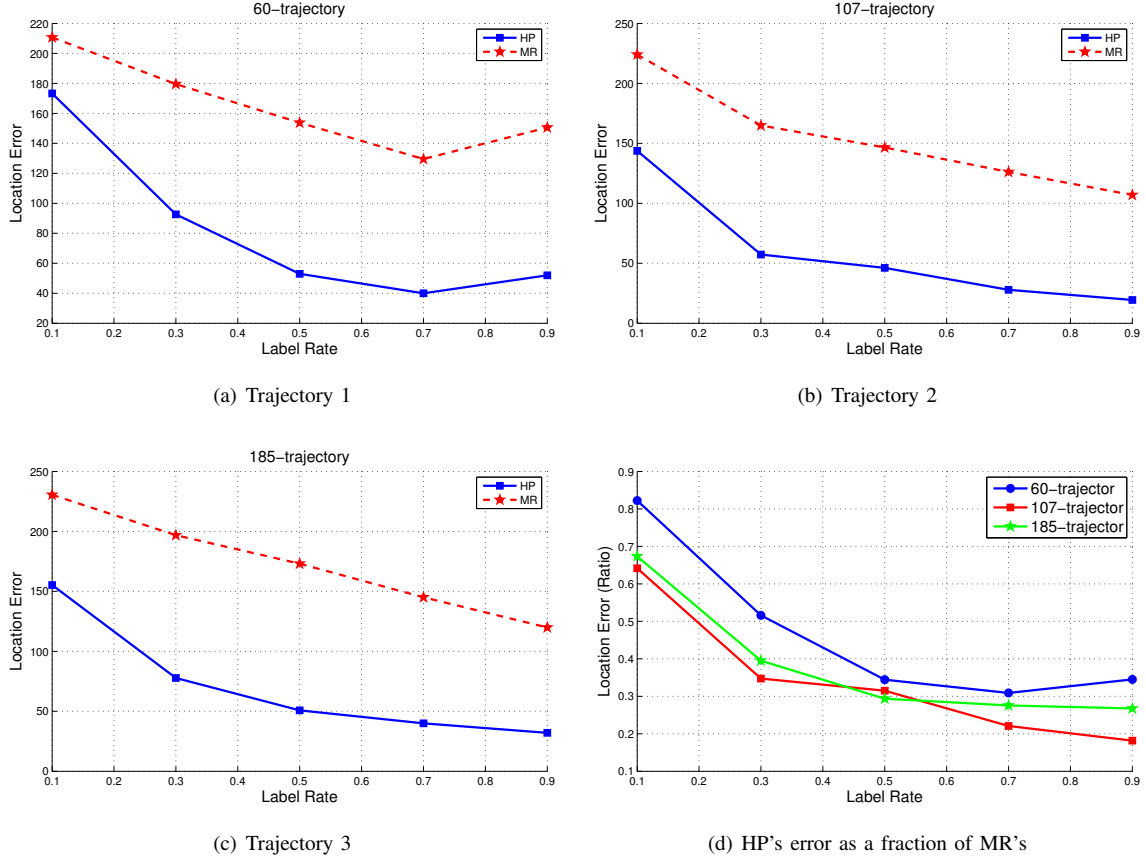


Fig. 2. Location error of MR vs. that of HP for the entire trajectory. The unit for the y-axis is 10cm.

The evaluation was conducted with a dataset collected in an experiment on the floor of our Computer Science department (Figure 1(a)). This dataset consists of 208 WiFi RSSI fingerprints at 208 locations sampled throughout this floor, respectively. There are in total 138 Wi-Fi access points and from those unreachable the corresponding RSSI is set to -100db. At each sample location, the corresponding fingerprint is the average of the RSSIs observed at this location. RSSI was measured by a person carrying an Android phone in no particular heading direction.

We consider three trajectories shown in Figure 1(b), Figure 1(c), and Figure 1(d), each being a path connecting sample locations. For each trajectory, the labeling status of each point is determined based on a label rate $p_l \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ and, for each choice of p_l , the results are averaged over five random runs. We set $\gamma = 1$ for the weight function. For each case of study (sample trajectory, same label rate, same random run), we perform a cross-validation procedure to choose the best regularization coefficient λ_S for MR; i.e., that results in the best error. Similarly we have a separate cross-validation procedure to choose the coefficient λ_T for HP. The range of possible coefficient values in the cross-validation is $\{10^{-7}, 10^{-6}, \dots, 10^{-1}, 1, 10\}$, representing ten different scales. The comparison is between HP using its best coefficient λ_T versus MR using its best coefficient λ_S .

Figure 2(a), Figure 2(b), and Figure 2(c) show the location errors of MR and HP for each of the three trajectories, respectively. It is expected that the error should decrease as the label rate increases. What is more noticeable, however, is the obvious superiority of HP's accuracy compared to MR's. For example, when only 50% of the fingerprints are labeled, for all three trajectories, HP has an error of roughly 5m while MR's error is more than 15m. In most cases, compared to MR, HP consistently cuts the error down by a factor of 1.5 times or more. This can be observed in Figure 2(d) showing the error of HP as a fraction of the error of MR.

Figure 3 shows the evolution of location error of each approach over the time, here showing the result for the 185-fingerprint trajectory for different cases of label rate p_l . It is observed again that throughout the travel path HP is obviously better than MR by a large margin, and this is regardless of whether the label rate is small (Figure 3(a)) or large (Figure 3(e)). Figure 3(f) plots the result averaging over all five cases of label rate. Another observation favoring HP is that its error converges to a stable value quickly as more fingerprints are observed whereas MR's error keeps increasing before showing any sign of convergence. This observation is clearer for the case of large label rates ($p_l \geq 0.3$) than that for the case of small label rate ($p_l = 0.1$).

Figure 4 draws the estimated trajectories resulted from

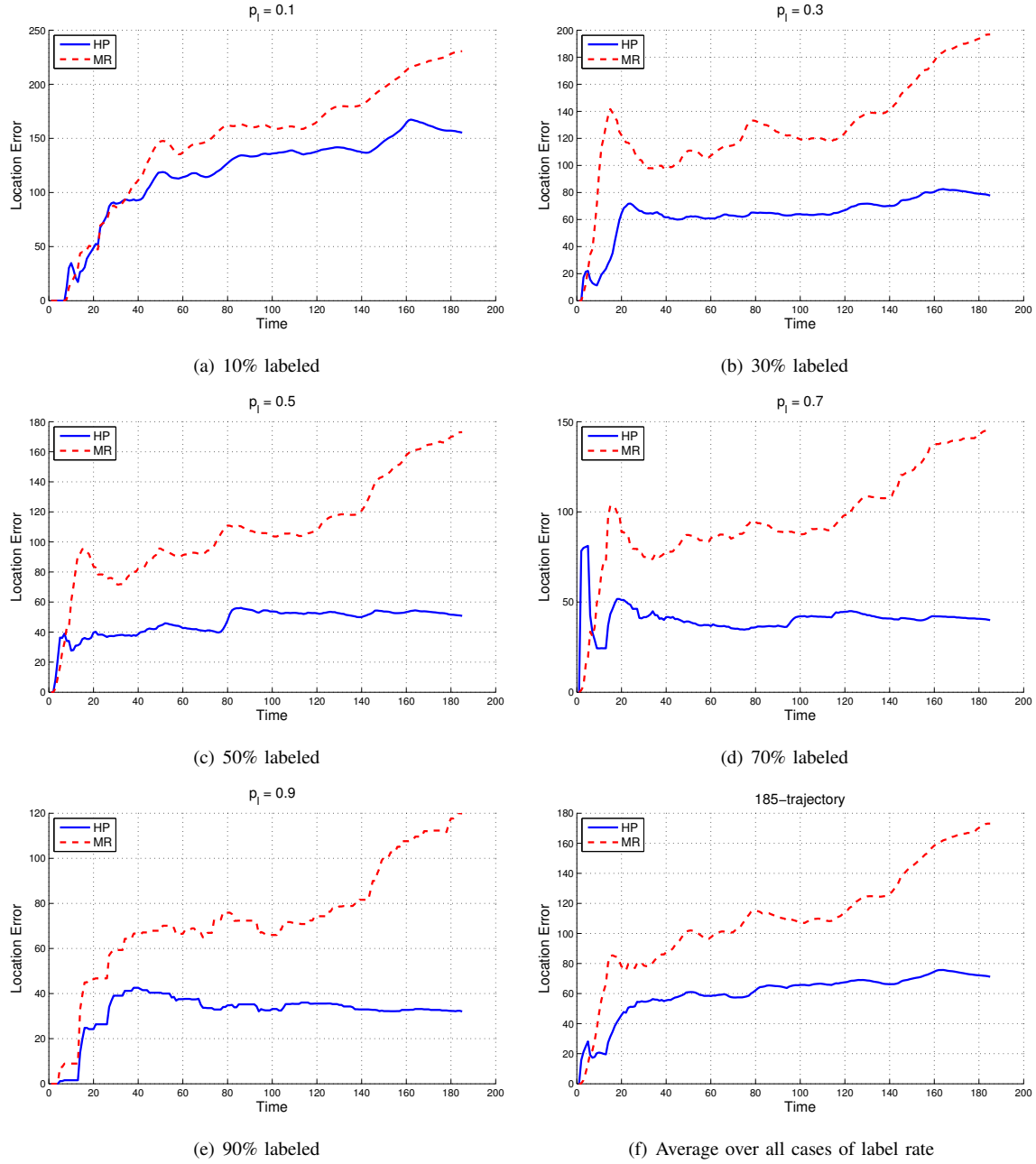


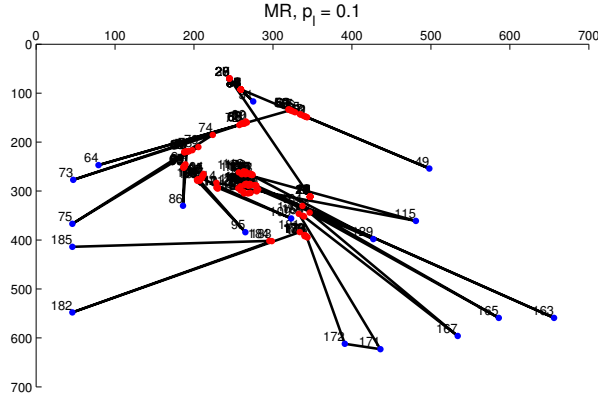
Fig. 3. Evolution of location error over time for the 185-trajectory. The unit for the y-axis is 10cm. Not only HP is more accurate than MR throughout the trajectory, HP does not get worse as quickly as MR does. Instead, HP converges to a stable error.

applying MR and HP to the 185-fingerprint sequence for three cases of label rates: 10% of the fingerprints are labeled ($p_l = 0.1$), 50% labeled ($p_l = 0.5$), and 90% labeled ($p_l = 0.9$). Here, we show the location estimated for a fingerprint instantly at the time it is observed. The first point is always put at the center because in the sequence generated it happens to be unlabeled and there is no labeled fingerprint available for learning. As can be seen in this figure, in all cases of label rate, HP's trajectory resembles the ground-truth trajectory more closely than MR does. Even in the case only 50% of the fingerprints are labeled, HP results in a trajectory

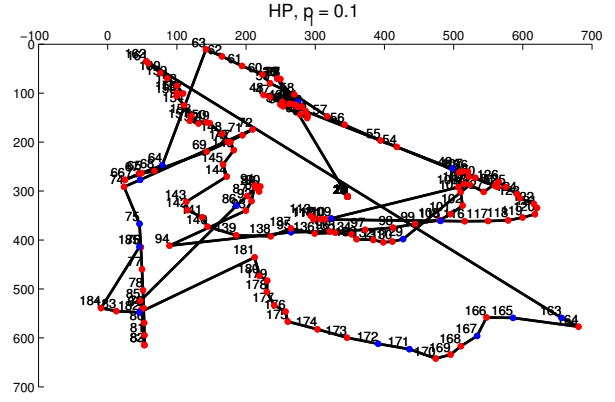
(Figure 4(d)) comparable to the trajectory produced by MR for the case 90% labeled (Figure 4(e)). It is noted that, after all the fingerprints in the sequence are observed, we can use the latest location estimator to obtain better estimates for all the unlabeled fingerprints in the past, including, for example, the first fingerprint. These estimates are useful if there is a need for a posterior fix of the trajectory.

V. CONCLUSIONS

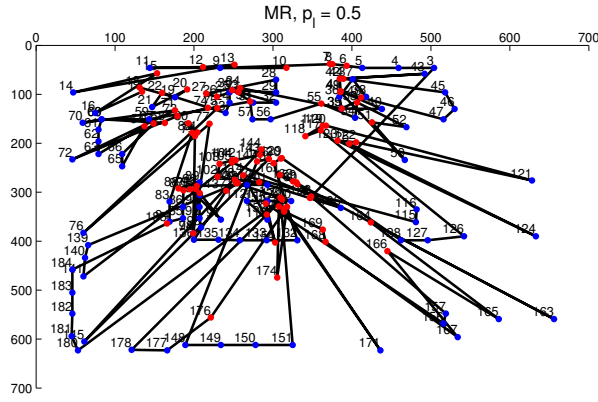
We have shown convincingly that temporal smoothing is an important property we should take into account for fingerprint-



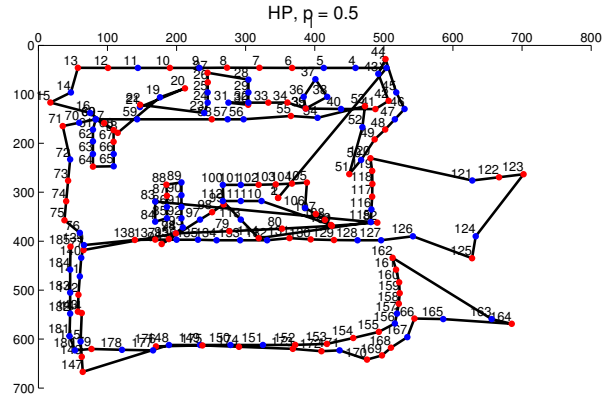
(a) MR: 10% labeled



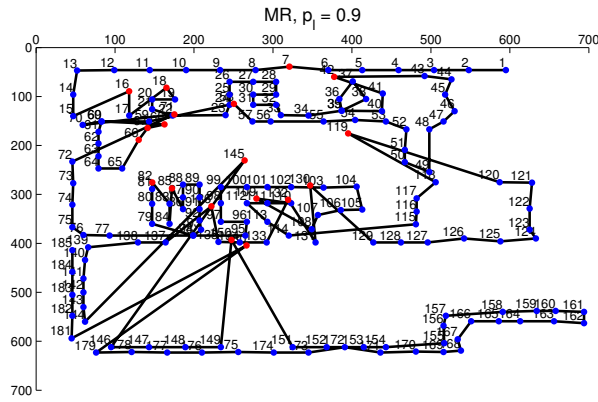
(b) HP: 10% labeled



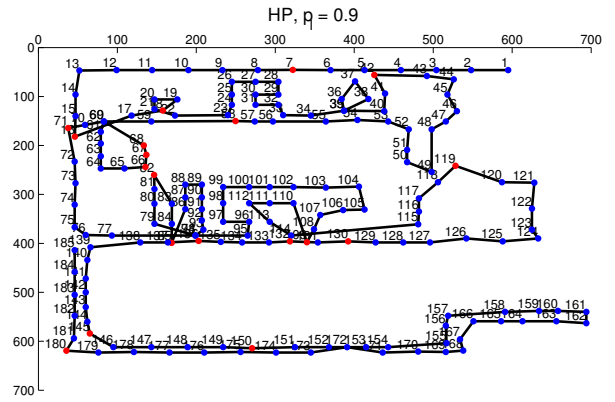
(c) MR: 50% labeled



(d) HP: 50% labeled



(e) MR: 90% labeled



(f) HP: 90% labeled

Fig. 4. Drawing of the estimated 185-trajectory. Red-colored points are location estimates for unlabeled fingerprints and blue-colored points are the ground-truth locations of the labeled fingerprints. The numbers represent the ID of the fingerprints sorted in time of measurement.

based location tracking. We have investigated the use of Hodrick-Prescott filtering as a way to integrate temporal smoothness in the location estimation. Not only computationally faster, but this approach has consistently been shown in our evaluation study to be more accurate than the conventionally used manifold regularization approach which factors in only the spatial smoothness property. For future work, we plan to investigate online/streaming algorithms that can locate each fingerprint in real time as it arrives without requiring to store the entire set of previously observed fingerprints. We also want to evaluate with more comprehensive experiments over larger time and spatial scales.

ACKNOWLEDGMENTS

This work was supported by the NSF award CNS-1116430. Any opinions, findings and conclusions or recommendations expressed in this material are ours and do not necessarily reflect those of the NSF.

REFERENCES

- [1] P. Bahl and V. N. Padmanabhan, "Radar: An in-building rf-based user location and tracking system," in *INFOCOM*, 2000, pp. 775–784.
- [2] Y. Chen, D. Lymberopoulos, J. Liu, and B. Priyantha, "Fm-based indoor localization," in *Proceedings of the 10th international conference on Mobile systems, applications, and services*, ser. MobiSys '12. New York, NY, USA: ACM, 2012, pp. 169–182. [Online]. Available: <http://doi.acm.org/10.1145/2307636.2307653>
- [3] A. Varshavsky, E. de Lara, J. Hightower, A. LaMarca, and V. Otsason, "Gsm indoor localization," *Pervasive Mob. Comput.*, vol. 3, no. 6, pp. 698–720, Dec. 2007. [Online]. Available: <http://dx.doi.org/10.1016/j.pmcj.2007.07.004>
- [4] S. P. Tarzia, P. A. Dinda, R. P. Dick, and G. Memik, "Indoor localization without infrastructure using the acoustic background spectrum," in *Proceedings of the 9th international conference on Mobile systems, applications, and services*, ser. MobiSys '11. New York, NY, USA: ACM, 2011, pp. 155–168. [Online]. Available: <http://doi.acm.org/10.1145/1999995.2000011>
- [5] J. Chung, M. Donahoe, C. Schmandt, I.-J. Kim, P. Razavai, and M. Wiseman, "Indoor location sensing using geo-magnetism," in *Proceedings of the 9th international conference on Mobile systems, applications, and services*, ser. MobiSys '11. New York, NY, USA: ACM, 2011, pp. 141–154. [Online]. Available: <http://doi.acm.org/10.1145/1999995.2000010>
- [6] J. J. Pan, Q. Yang, H. Chang, and D. Y. Yeung, "A manifold regularization approach to calibration reduction for sensor-network based tracking," in *Proceedings of the Twenty-First National Conference on Artificial Intelligence*, Boston, United States, 2006, pp. 988–993.
- [7] J. J. Pan and Q. Yang, "Co-localization from labeled and unlabeled data using graph laplacian," in *Proceedings of the Twentieth International Joint Conference on Artificial Intelligence*, Hyderabad, India, 2007, pp. 2166–2171.
- [8] T. Pulkkinen, T. Roos, and P. Myllymäki, "Semi-supervised learning for wlan positioning," in *Proceedings of the 21th international conference on Artificial neural networks - Volume Part I*, ser. ICANN'11. Berlin, Heidelberg: Springer-Verlag, 2011, pp. 355–362. [Online]. Available: <http://dl.acm.org/citation.cfm?id=2029556.2029600>
- [9] Y. Jia Zhu, Z. liang Deng, and H. Ji, "Indoor localization via l1-graph regularized semi-supervised manifold learning," *The Journal of China Universities of Posts and Telecommunications*, vol. 19, no. 5, pp. 39 – 91, 2012. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S1005888511602987>
- [10] D. A. Tran and P. Truong, "Total variation regularization for training of indoor location fingerprints," in *ACM MOBICOM Workshop on Mission-Oriented Wireless Sensor Networking (ACM Misenet 2013)*, Miami, Sep 2013.
- [11] C. Wu, Z. Yang, Y. Liu, and W. Xi, "Will: Wireless indoor localization without site survey," *IEEE Transactions on Parallel and Distributed Systems*, vol. 24, no. 4, pp. 839–848, 2013.
- [12] O. Woodman and R. Harle, "Pedestrian localisation for indoor environments," in *Proceedings of the 10th International Conference on Ubiquitous Computing*, ser. UbiComp '08. New York, NY, USA: ACM, 2008, pp. 114–123. [Online]. Available: <http://doi.acm.org/10.1145/1409635.1409651>
- [13] L. Hu and D. Evans, "Localization for mobile sensor networks," in *Proceedings of the 10th Annual International Conference on Mobile Computing and Networking*, ser. MobiCom '04. New York, NY, USA: ACM, 2004, pp. 45–57. [Online]. Available: <http://doi.acm.org/10.1145/1023720.1023726>
- [14] A. Boukerche, H. A. B. F. Oliveira, E. F. Nakamura, and A. A. F. Loureiro, "Vehicular ad hoc networks: A new challenge for localization-based systems," *Comput. Commun.*, vol. 31, no. 12, pp. 2838–2849, Jul. 2008. [Online]. Available: <http://dx.doi.org/10.1016/j.comcom.2007.12.004>
- [15] I. Amundson and X. D. Koutsoukos, "A survey on localization for mobile wireless sensor networks," in *Proceedings of the 2Nd International Conference on Mobile Entity Localization and Tracking in GPS-less Environments*, ser. MELT'09. Berlin, Heidelberg: Springer-Verlag, 2009, pp. 235–254. [Online]. Available: <http://dl.acm.org/citation.cfm?id=1813141.1813162>
- [16] M. Belkin, P. Niyogi, and V. Sindhwani, "Manifold regularization: A geometric framework for learning from labeled and unlabeled examples," *J. Mach. Learn. Res.*, vol. 7, pp. 2399–2434, Dec. 2006. [Online]. Available: <http://dl.acm.org/citation.cfm?id=1248547.1248632>
- [17] R. J. Hodrick and E. C. Prescott, "Post-war u.s. business cycles: An empirical investigation," Northwestern University, Center for Mathematical Studies in Economics and Management Science, Discussion Papers 451, 1981. [Online]. Available: <http://EconPapers.repec.org/RePEc:nwu:cmsems:451>
- [18] D. A. Tran and T. Nguyen, "Localization in wireless sensor networks based on support vector machines," *IEEE Transactions on Parallel and Distributed Systems*, vol. 19, no. 7, pp. 981–994, July 2008.
- [19] M. Youssef and A. Agrawala, "The horus location determination system," *Wirel. Netw.*, vol. 14, no. 3, pp. 357–374, Jun. 2008. [Online]. Available: <http://dx.doi.org/10.1007/s11276-006-0725-7>
- [20] R. Want, A. Hopper, V. Falcão, and J. Gibbons, "The active badge location system," *ACM Trans. Inf. Syst.*, vol. 10, no. 1, pp. 91–102, Jan. 1992. [Online]. Available: <http://doi.acm.org/10.1145/128756.128759>
- [21] W. Zhang and M. Kavehrad, "A 2-d indoor localization system based on visible light led," in *Photonics Society Summer Topical Meeting Series, 2012 IEEE*, July 2012.
- [22] A. Savvides, C.-C. Han, and M. B. Strivastava, "Dynamic fine-grained localization in ad hoc networks of sensors," in *ACM International Conference on Mobile Computing and Networking (MOBICOM 2001)*, Rome, Italy, July 2001, pp. 166–179.
- [23] T. He, C. Huang, B. Blum, J. Stankovic, and T. Abdelzaher, "Range-free localization schemes in large scale sensor networks," in *ACM Conference on Mobile Computing and Networking*, 2003.
- [24] T. Roos, P. Myllymäki, H. Tirri, P. Misikangas, and J. Sievänen, "A probabilistic approach to WLAN user location estimation," *International Journal of Wireless Information Networks*, vol. 9, no. 3, pp. 155–164, July 2002. [Online]. Available: <http://dx.doi.org/10.1023/A:1016003126882>
- [25] C. Laoudias, D. G. Eliades, P. Kemppi, C. G. Panayiotou, and M. M. Polycarpou, "Indoor localization using neural networks with location fingerprints," in *Proceedings of the 19th International Conference on Artificial Neural Networks: Part II*, ser. ICANN '09. Berlin, Heidelberg: Springer-Verlag, 2009, pp. 954–963.
- [26] M. Brunato and R. Battiti, "Statistical learning theory for location fingerprinting in wireless lans," *Comput. Netw.*, vol. 47, no. 6, pp. 825–845, Apr. 2005. [Online]. Available: <http://dx.doi.org/10.1016/j.comnet.2004.09.004>
- [27] L. I. Rudin, S. Osher, and E. Fatemi, "Nonlinear total variation based noise removal algorithms," *Phys. D*, vol. 60, no. 1–4, pp. 259–268, Nov. 1992. [Online]. Available: [http://dx.doi.org/10.1016/0167-2789\(92\)90242-F](http://dx.doi.org/10.1016/0167-2789(92)90242-F)
- [28] S. Rallapalli, L. Qiu, Y. Zhang, and Y.-C. Chen, "Exploiting temporal stability and low-rank structure for localization in mobile networks," in *Proceedings of the sixteenth annual international conference on Mobile computing and networking*, ser. MobiCom '10. New York, NY, USA: ACM, 2010, pp. 161–172. [Online]. Available: <http://doi.acm.org/10.1145/1859995.1860015>