Joint Network Coding and Power Control for Cellular Radio Networks

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Abstract—We investigate the problem of optimizing power consumption and bandwidth usage in cellular radio networks. In this paper we consider a wireless broadcast network, organized in cells, in which each transmitter wants to deliver the same amount of data to all receivers inside its cell. Traditionally, the transmitter in each cell radiates a fixed power at which the receiver at the border of the cell can receive, in theory, a signal whose power is greater than its required threshold and satisfies condition of signal to interference ratio (SIR). When a receiver in a cell receives an error packet, the transmitter in that cell will retransmit until the receiver receives it successfully. Instead in network coding approach, the transmitter will store index of the packets, and then when a receiver in a cell receives an error packet, the transmitter in that cell will retransmit until the receiver receives it. The goal of this paper is to provide an analysis on the power consumption and bandwidth utilization of network coding techniques at the signal layer. Especially, we propose a novel technique in which network coding and adaptive power control are joined together to improve network performance. The simulation results show that our proposed technique substantially reduces power consumption while increases the network throughput compared with the traditional transmission technique, Automatic Repeat reQuest (ARQ).

I. INTRODUCTION
Power is known as a critical parameter in wireless network due to the limited battery life. Moreover, propagation medium in wireless network is susceptible to impairments such as path loss, multi-path propagation and interferences. Therefore, to limit multi-user interference, and hence, improve quality of service, power control is used as an important technique.

Traditionally, in cellular radio networks, transmitter of each cell radiates a fixed power at which it can cover the cell area. The transmission power is calculated so that if existing a receiver at the border of the cell, it can still receive a signal satisfying conditions of recovery of the original data. When a receiver receives an error packet, the transmitter will retransmit that packet until it is received correctly. However, this approach has been shown to be suboptimal in terms of network resource utilization, especially, when applied to wireless networks such as ad hoc or sensor networks.

Recently, a new approach, network coding (NC), appeared in 2000 with the pioneering work of Ahlswede [1], has made the existing problems of the traditional systems can be utilized to ease while providing better network performance. The main principle of network coding is to have the intermediate nodes inside the network combine different input flows before sending out. By doing that the network throughput can be improved substantially. In particular, Ahlswede has shown that the network capacity for a single-source multicast network can be achieved by network coding.

Based on this approach Tran et al. [2] proposed a joint network and channel coding technique to increase the bandwidth efficiency of broadcast and unicast sessions in a single-hop wireless network such as Wireless Local Area Networks (WLAN). In this approach, the AP (Access Point) maintains a queue of lost packets, and combine different lost packets from different receivers in such a way to allow multiple receivers to recover their lost packets simultaneously with one transmission from the AP. In this paper, we first extend the previous results by providing an analysis on network performances at the signal level. Moreover, we propose a novel technique which joins network coding and adaptive power control to reduce power consumption and increase bandwidth utilization for a multiple-cell wireless network. Our contributions include (a) some analytical results on the power consumption and bandwidth efficiency at the physical level for a cellular radio network; (b) a novel technique which joins network coding and power control to improve network power and bandwidth usage.

The organization of our paper is as follows. We first discuss some related work in Section II. In Section III, we describe the system model and assumptions. In Section IV, we provide some theoretical analysis on the transmission channel, characterize packet error probability, power consumption and bandwidth utilization. Simulation results and discussions are provided in Section V. Finally, we conclude with few remarks in Section VI.

II. RELATED WORK
The motivation of our paper is from the idea of opportunistic network coding proposed by Katti et al. [3]. In [3], by allowing each node in the network to snoop on the medium, learn the status of its neighbors, detect coding opportunities, and code as long as the recipients can decode, the network bandwidth can be substantially improved over the current techniques. Based on that approach, we proposed a novel technique which joins network coding and adaptive power control. The main idea of our proposed technique is adaptively control transmission power to create chances of combining error packets for retransmission phase. The error packets are not only considered in a single cell but also in multiple cells. This means that network coding is not opportunistic but intended.

Our work is also related to the wireless model in optimizing consumption energy proposed by Lun et al. [4]. In this, the authors have proved that the problem of minimum-energy multicast in wireless networks can be solved exactly in polynomial time when using network coding. Also, Y. Wu et al. [5] has shown that by applying network coding in a multiple mobile ad hoc network one can minimize energy consumption.

In addition, in the works on multi-hop wireless network with multicast sessions, Li et al. [6][7] have proved that network coding can provide marginal benefits over the approaches that do not use network coding. Also, Lun et al. [8] shows a capacity-approaching coding scheme for unicast or multicast over lossy packet networks in which all nodes perform opportunistic coding by constructing encoded packet with random linear combinations of previously received packets. There is also other literature on network coding and power control schemes for wireless networks [9][10].

III. PROBLEM FORMULATION
We first begin with a set of assumptions on channel model and protocols.

A. Assumptions

1) Network is organized in cells, each cell has a transmitter and all the transmitters connected with each other by high speed links.

2) Data is assumed to be sent in packets, and each packet is sent in a time slot of fixed duration.

3) Data is assumed to be sent in packets, and each packet is sent in a time slot of fixed duration.
cells of the network. The red dots denote receivers with error packets. The number near to each receiver denotes error packet index. Using ARQ or NC in transmission in one cell is suboptimal in terms of power consumption and bandwidth usage compared with applying network coding for multiple cells of the network.

3) The source assumes to know which packet from which receiver is lost. This can be accomplished through the use of positive and negative acknowledgments (ACK/NAKs). For simplicity, we assume all the ACK/NAKs are instantaneous, i.e., the source knows (a) whether or not a packet is lost and (b) identity of the receiver with the lost packet instantly. This implicitly assumes that ACK/NAKs are never lost. This assumption is not critical as we can easily incorporate the delay and bandwidth used by ACK/NAKs into the analysis.

4) The transmitters are located at the center of each cell and use omnidirectional antenna. The transmitter has an ability to adjust its transmission power from $P_{min}$ to $P_{max}$, corresponding to a radius from $r_{min}$ to $r_{max}$. We assume that all cells have the same number of receivers with the same sensitivity. The receivers also have ability to detect the pilot tone of the transmitted signal from different transmitters at different frequencies. The receivers are uniformly distributed over cell area.

5) The transmission channel follows the log-normal propagation model. A packet is lost if the received power is less than the receiver's sensitive threshold or $SIR$ is less than a margin threshold. Furthermore, the packet loss at receivers are uncorrelated. This model is clearly insufficient to describe many real-world scenarios. One can develop a more accurate model, albeit complicate analysis. Under these settings, the question we address in this paper is: What is the optimal power consumed by all the transmitters to deliver an amount of information to all receivers in the network? On the other hand, given a network topology and a number of information packets, we want to find out a technique which uses minimal energy and bandwidth to deliver all the intended information to the receivers. For the sake of clarity, the network under investigation in this paper consists of $K=7$ cells with its topology is illustrated in Fig. 1. A generic case of radio cellular network having more than 7 cells is considered as a future work. Before proceeding to the details of analysis, let us provide some definitions and network protocols which will be used in the rest of the paper.

B. Transmission Schemes

1) Auto Repeat reQuest (ARQ): Assume the transmitters want to deliver $M$ packets to all the receivers inside the network. To send a packet, the transmitter transmits a power $P_0$ so that the transmission signal can reach the cell border. For ARQ scheme, if one of the receivers inside the cell receives a corrupted packet, the transmitter retransmits the error packet until the receiver receives it successfully.

2) Network coding (NC): $M$ packets are transmitted by the first $M$ time slots, with transmission power $P_0$. The feedback NAKs from receivers are stored in the transmitters buffer memory. Since the transmission protocol is identical in all cells, for the sake of clarity, let us consider a cell consisting of one transmitter and three receivers $R_1, R_2$ and $R_3$. After the first $M$ time slots, the transmitter will consider combining error packets for retransmission based on indexes of error packets of receivers inside the cell. For example, from the error pattern of receivers as shown in Fig. 2, the transmitter combines packet $b_1, b_2$ and $b_3$ by XORing as $b_1 \oplus b_2 \oplus b_3$. Now with one retransmission, all the receivers can recover their own useful information, assuming that they received the combined packet successfully. For instance, the receiver $R_1$ can recover packet $b_1$ as $(b_1 \oplus b_2) \oplus (b_1 \oplus b_3)$. Similarly, receiver $R_2$ and $R_3$ recover packets $b_2$ and $b_3$ as $(b_1 \oplus b_2) \oplus (b_1 \oplus b_3)$ and $(b_1 \oplus b_2) \oplus (b_1 \oplus b_3)$, respectively. Other error packets will be retransmitted as $b_1 \oplus b_2 \oplus b_3$. One should notice that the transmitter can dynamically change the combined packets based on what the receivers have received. For example, after retransmitting packet $b_2$, it is still lost at receiver $R_2$, but received correctly at $R_1$ and $R_3$. In this case, in stead of retransmitting packet $b_2$, the transmitter transmits a combined packet as $b_1 \oplus b_3 \oplus b_3$. Clearly, by dynamically changing the packets in the combined packet, the transmitter can maximize possible useful information in each transmission.

To have a sense of how much energy will be used in this technique, let us describe it in the network scenario as shown in Fig. 1. After $M$ transmissions, receivers with error packets are denoted as the red dots. Now the transmitter in each cell will consider combining error packets for retransmission phase. For example, in cell 1, the transmitter will combine packet $b_1$ and $b_2$ for retransmission. The transmission power for the combined packet is $P_0$. Similarly, the transmitters at cell 2, 3, 4, 5, 6 and 7 will combine error packets, if possible, for retransmission until all the receivers receive their needed packets successfully. Assume that each transmitter needs one transmission to deliver an error packet. Therefore, the total number of retransmissions used by the transmitters in the network is 10. This means that the energy consumed by transmitters for retransmission phase is $10P_0$. 3) Joint network coding and power control (NCPC): In this scheme, we propose a novel technique that joins network coding and power control for retransmission phase. Let us consider the same network as shown in Fig. 1. Similarly as in NC scheme, all the transmitters use their first $M$ time slots to transmit $M$ original packets sequentially. Since a transmitter knows the error packets index of its receivers, through an information exchange, all other transmitters can also have this information. Moreover, we assume that the transmitter can roughly estimates locations of the receivers. With the network scenario as shown in Fig. 1, the transmitter at the center cell can transmit a combined packet as $b_2 \oplus b_3 \oplus b_4 \oplus b_5$ at a higher transmission power, called $P_0$, to cover not only the receivers inside its cell but also the receivers of the neighbor cells. Assume that the transmitter of the center cell needs one transmission to successfully deliver the combined packet to all the receivers having error

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Footnote:
1 How to estimate a receiver locations is out of the scope of this paper, one can refer to many other literature for details.
packets. Clearly, the receivers with error packets can recover their needed information by properly operating XOR. The next combined packet is $b_4 \oplus b_5$ and the needed power for transmission it is also $P_{in}$ since this combined packet needs to reach to both receivers in cell 2 and 4. Simultaneously, transmitters in cell 1, 3, and 6 decrease their transmission powers to retransmit non-combined packets. Transmission power for these error packets is denoted as $P_{de}$. With NCPC, total energy consumed by all transmitters in the network is $2P_{in} + 4P_{de}$. Intuitively, when positions of receivers with error packets near to border of the center cell, the consumption power used by NCPC scheme is much less than that of ARQ or NC schemes.

Definition 3.1: Energy Per Packet (EPP): The total consumption energy used by all transmitters in the network divided by total number of original information packets.

Let consider the previous example, ARQ and NC schemes need 12 and 10 retransmissions to deliver all error packets successfully. Therefore, EPPs of ARQ and NC schemes for delivering $M$ original information packets are given by

$$EPP_{ARQ} = \frac{(7M + 12)P_0}{M} \text{ (energy unit per packet)} \quad (1)$$

and

$$EPP_{NC} = \frac{(7M + 10)P_0}{M} \text{ (energy unit per packet)} \quad (2)$$

Similarly, EPP of NCPC scheme is

$$EPP_{NCPC} = \frac{7MP_0 + 2P_{in} + 4P_{de}}{M} \text{ (energy unit per packet)} \quad (3)$$

Based on the EPP definition, a scheme is better than others if its EPP is smaller.

Next, the number of time slots used to deliver all intended packets to the receivers will be considered. We assume a fixed underlying physical bandwidth, and therefore the time required to successfully transmit all the packets to the intended receivers can be characterized by ratio of the number of data packets to the actual transmitted packets in the network. Based on this, all schemes under investigation will use the following definition of the bandwidth efficiency as the evaluating metric.

Definition 3.2: Bandwidth efficiency (BWE): Ratio of the ideal needed time slots to the actual used time slots to deliver all original information packets.

Again, recall the previous example, ARQ and NC schemes need $M$ time slots to deliver the original packets. Then in ARQ scheme, the transmitters at cell 0, 5 and 6 need one more time slot for retransmission phase, while those of cells 1, 2, 4 and 3 are 2 and 3, respectively. Therefore, BWE of ARQ scheme is

$$BWE_{ARQ} = \frac{M}{M + 3} \quad (4)$$

In NC scheme, transmitters at cells 1, 5, 6 and 0 need one more time slot to deliver the error packets while those of cell 2, 3 and 4 are two more time slots. Hence, BWE of NC scheme is

$$BWE = \frac{M}{M + 2} \quad (5)$$

Similarly, NCPC scheme needs two more time slots for the transmitter of the center cell and the transmitters in cells 1, 3 and 6 to deliver the combined and non-combined packets respectively. Therefore the BWE of NCPC scheme is given by

$$BWE = \frac{M}{M + 2} \quad (6)$$

Therefore, BWE of a scheme is always less than or equal to 1. BWE equals to 1 when all the information packets are transmitted without error; this is an ideal case.

IV. NETWORK PERFORMANCE ANALYSIS

A. Propagation model

In wireless communications, the average received power at a receiver depends on characteristics of the channel and distance between the transmitter and receiver. The loss in signal strength due to the distance factor is known as the propagation path loss [11]. Free space path loss is a function of transmission frequency, $f$, distance between transmitter and receiver, $d$, and characteristics of the propagation environment, $\alpha$. We have

$$P_r^0 = \frac{P_t}{(4\pi d^2 f)^\alpha} \quad (7)$$

where, $P_t$ is transmission power, $P_r^0$ is received power. Value of $\alpha$ varies from 2 to 6 [11].

However, this model could be inaccurate since in reality the received power level may show significant variation around the mean power. Therefore, a more realistic radio model wireless communications has been used in the paper, that is log-normal shadowing model [12]. Let $P_r$ denote received power at a distance $r$ from the transmitter. According to the transmission model, values of $P_r$ are normally distributed around the logarithm value of the average power received at distance $r$. That is

$$10 \log_{10}(P_r) \sim N(10 \log_{10}(P_{avg}^r), \sigma^2) \quad (8)$$

or expressed in dBm

$$P_r \sim N(P_{avg}^r, \sigma^2) \quad (9)$$

where, $P_{avg}^r$ is the average power received at distance r, and $\sigma^2$ is the variance of the shadowing in dB. $P_{avg}^r$ is given by

$$P_{avg}^r(dBm) = P_0(dBm) - 10 \times \alpha \log_{10} \left( \frac{r}{r_0} \right) \quad (10)$$

where, $P_0$ is received power in dBm at a close-in distance $r_0$. The reference distance $r_0$ is chosen to be in the far-field of the antenna at which the propagation can be considered as that of free-space. Typically, $r_0$ is chosen to be 1(m) for indoor environments and 1(km) for outdoor environments. $P_0$ is determined by the following equation

$$P_0 = 10 \log_{10} \left( \frac{4\pi r_0^2}{\lambda} \right) \quad (11)$$

where, $\lambda$ is wave length of the transmission signal.

We assume that the reception of a radio signal at a receiver $j$ correct if and only if

- Received signal is at least equal to receiver’s sensitive threshold,
  $$P_j^r \geq \beta_j \quad (12)$$

- Signal-to-interference-and-noise ratio, SIR, is at least equal to a required margin $\gamma_j$
  $$SIR_j = \frac{P_{ij}}{N_j + \sum_{k=1, k \neq i}^k P_{kj}} \geq \gamma_j \quad (13)$$

where, $P_{ij}$ is received power at receiver $j$ when $i$ is the transmitter, $N_j$ is thermal noise of receiver $j$ and $k$ is the total simultaneous active transmitters.
B. Power and Bandwidth Computation

Recall that the number of receivers in a cell is \( N \), and the number of transmission packets is \( M \). In this part we will derive analysis on average energy and bandwidth consumption used by the transmitters in different schemes to deliver the intended data to the receivers. Moreover, for the sake of clarity, let us assume that \( \beta_j \) and \( \gamma_j \) are given and the same at all receivers.

1) ARQ scheme

In this scheme, transmitter sends a packet to receivers inside a cell using a transmission power \( P_0 \). When a receiver received a corrupted packet, the transmitter retransmits that packet until it is received correctly. Notice that, transmitters of several cells can transmit data simultaneously. Let \( P_{xy}^{ij} \) denote the probability of success delivery a packet from transmitter \( X \) to receiver \( Y \). Therefore, the probability that the receiver \( j \) receives a packet from transmitter \( i \) successfully is

\[
P_s^{ij} = P(P_r^i \geq \beta_j, P(SIR_{ij} \geq \gamma_j))
\]

Assume that received signals at receivers are independent. Hence, the probability that transmitter \( i \) successfully sends a packet to \( N \) receivers inside its cell is given by

\[
p_s^i = \prod_{j=1}^{N} p_s^{ij}
\]

It follows that the total energy consumed by all the transmitters to deliver \( M \) packets to all receivers in a network of \( K \) cells is

\[
E_T = \sum_{i=1}^{K} \frac{M \times P_0}{p_s^i}
\]

or in terms of EPP

\[
EPP_{ARQ} = \sum_{i=1}^{K} \frac{P_0}{p_s^i}
\]

Bandwidth efficiency of ARQ scheme is calculated by the maximum number of time slots used in a cell. Therefore,

\[
BWE_{ARQ} = \frac{1}{\max_{i \in \{1, \ldots, K\}} \left\{ \frac{1}{p_s^i} \right\}}
\]

2) NC scheme

In NC scheme, we use the first \( M \) time slots to transmit the original data. Then, based on indexes of error packets in the buffer memory, the transmitter will consider combine error packets for retransmission phase. One can refer to the Fig. 2 for how to create a combined packet. Similarly as in ARQ scheme, to send a packet the transmitter uses a fixed transmission power \( P_0 \). Before proceeding to calculate EPP and BWQ of NC scheme, we present the following theorem which give us the expected number of transmissions needed to successfully deliver a packet to all receivers inside the network. Theorem 4.1: The expected number of transmissions to successfully deliver a packet to all receivers inside the network when \( M \) large enough is

\[
\eta = \sum_{i=1}^{K} \frac{1}{\min_{j \in \{1, \ldots, N\}} \left\{ \frac{1}{p_s^i} \right\}}
\]

Proof: We begin with a simple case of two receivers. Without loss of generality, let us assume that the probability of successfully receiving a packet at receiver \( R_1 \) is greater than that of receiver \( R_2 \), \( P_s^{11} \geq P_s^{12} \). As discussed in Section III-B, the combined packets in NC scheme are dynamically formed based on the feedback from the receivers. If a combined packet is correctly received at some receivers, but not at others, a new combined packet is created to ensure that all the receivers receive useful information. This implies that after a long run, the number of losses will be dominated by the number of losses at the receiver with the greatest error probability \( R_2 \). Therefore, the total number of transmissions to successfully deliver \( M \) packets to two receivers equals to the number of transmissions to successfully deliver \( M \) packets to \( R_2 \) alone, i.e. \( \frac{M}{P_s^{22}} \) or \( \frac{M}{\min\{P_s^{21}, P_s^{22}\}} \). Without much difficulty, we can generalize this result to the network with \( N \) receivers

\[
n = \frac{M}{\min_{j \in \{1, \ldots, N\}} \left\{ \frac{1}{p_s^j} \right\}}
\]

Therefore, the expected number of required transmissions to deliver a successful packet to all receivers in a cell is

\[
\eta = \frac{n}{M} = \frac{1}{\min_{j \in \{1, \ldots, N\}} \left\{ \frac{1}{p_s^j} \right\}}
\]

For a network consisting of \( K \) cells, the average number of transmissions required to transmit a packet successfully to all receivers in the network is

\[
\eta = \sum_{i=1}^{K} \frac{1}{\min_{j \in \{1, \ldots, N\}} \left\{ \frac{1}{p_s^j} \right\}}
\]

From the result of Theorem (4.1) we have EPP in NC scheme is given by

\[
EPP_{NC} = \sum_{i=1}^{K} \min_{j \in \{1, \ldots, N\}} \left\{ \frac{P_0}{p_s^j} \right\}
\]

The bandwidth efficiency BWE is

\[
BWE_{NC} = \frac{1}{\max_{i \in \{1, \ldots, K\}} \left\{ \frac{1}{\min_{j \in \{1, \ldots, N\}} \left\{ \frac{1}{p_s^j} \right\}} \right\}}
\]

3) NCPC scheme

Assumed that the transmitter at the center cell increases its transmission power to cover an area with a radius \( r_{in} \). By using geometry one can prove that the extension area that the center transmitter overlaps with a neighbor cell, as shown in Fig. 3(a), is given by

\[
S_E = \arccos \left( \frac{h_1}{r_0} \right) r_0^2 - h_1 \sqrt{r_0^2 - h_1^2} + \arccos \left( \frac{H}{r_{in}} \right) r_{in}^2 - H \sqrt{r_{in}^2 - H^2} - 2 \right) \arccos \left( \frac{h_1}{r_0} \right) r_0^2 - h_1 \sqrt{r_0^2 - h_1^2}
\]

where,

\[
\Delta r = r_{in} - r_0, \quad H = r_0 + \frac{\Delta r(2r_0 + \Delta r)}{4r_0}, \quad h_1 = r_0 + \frac{\Delta r(2r_0 + \Delta r)}{4r_0}
\]
Since the receivers are uniformly distributed over a cell, the expected number of receivers of a neighbor cell \( i \) lies inside the extension area is given by
\[
N^i_E = \frac{N_i S_E}{\pi r_0^2}
\]  
(26)

Therefore, the total number of receivers of neighbor cells lies inside the extension area is given as follows
\[
N^0_E = \sum_{i=1}^{6} N^i_E = \left[ \frac{6 N S_E}{\pi r_0^2} \right]
\]  
(27)

Based on the transmission protocol designed for NCPC scheme as described in Section III-B, after the first \( M \) time slots, the transmitter at the center cell will consider increasing its transmission power to retransmit the combined packets. If that is the case, the network now is consisting of one larger cell at the center and six other smaller cells, as shown in Fig. 3(b). Hence, more receivers have been added into the new center cell from the neighbor cells. Notice that increasing transmission power is implemented only during retransmission phase. Therefore, receivers without error packets should not be taken into account. Let \( P^i_j \) denote packet error probability at receiver \( j \) when \( i \) is the transmitter. From (14), we have
\[
P^i_j = 1 - P^{ij}_s
\]  
(28)

We only consider increasing transmission power at the center cell when there exists a receiver inside the extension area and at least one receiver inside the center cell or there are more than two receivers inside the extension area having error packets. For the sake of simplicity, let us assume that the receivers with error packets are located either inside the extension area or the reduced radius circles, or the center cell. Actually, the probability that a receiver with error packets is outside this area is very small and can be negligible. The network topology is now reconfigured with one greater cell at the center and six smaller edge cells, as illustrated in Fig. 3(b). The expected number of receivers lying inside a reduced radius circle is calculated as
\[
N_d = \left\lfloor \left( \frac{r_0 - \Delta r}{r_0} \right)^2 \right\rfloor
\]  
(29)

Let \( P_L(k) \) denote the probability that \( k \) out of \( L \) receivers having error packets in cell \( i \). We have
\[
P_L(k) = \binom{L}{k} \left( P^{ij}_s \right)^k \left( 1 - P^{ij}_s \right)^{L-k}
\]  
(30)

Therefore, the average number of receivers with error packets inside a reduced radius circle is given by
\[
N_{in} = \sum_{k=0}^{N_d} k P_{N_k}(k)
\]  
(31)

and the expected number of receivers with error packets inside the extension area is
\[
N_{ec} = \sum_{k=0}^{N_E} k P_{N_k}(k)
\]  
(32)

Moreover, the number of receivers with error packets inside the center cell is given by
\[
N_c = \sum_{k=0}^{N} k P_{N_k}(k)
\]  
(33)

Combining (32) and (33) gives us the total number of receivers with error packets inside the increasing radius circle
\[
N_{ec} = N_c + N_e
\]  
(34)

Applying the result of Theorem (4.1) to the reduced radius cells gives us the expected number of transmissions to successfully deliver a packet
\[
\eta_{in} = \sum_{i=1}^{6} \min_{j \in \{1,\ldots,N_{in} \}} \{ P^{ij}_s \}
\]  
(35)

Similarly, applying to the increasing radius cell yields
\[
\eta_{ec} = \frac{1}{\min_{j \in \{1,\ldots,N_{ec} \}} \{ P^{ij}_s \}}
\]  
(36)

where \( P^{ij}_s \) denotes the probability that receiver \( j \) received a packet successfully when the receiver is at the center cell. From (35) and (36) we have EPP used in NCPC scheme is given by
\[
EPP = (K + \eta_{in} + \eta_{ec}) P_0
\]  
(37)

and the bandwidth efficiency BWE is
\[
BWE = \max \left\{ \max_{i \in \{1,\ldots \}} \min_{j \in \{1,\ldots,N_{in} \}} \{ P^{ij}_s \}, \min_{j \in \{1,\ldots,N_{ec} \}} \{ P^{ij}_s \} \right\}
\]

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, we provide simulation results on network performance in terms of power and bandwidth consumption of different transmission schemes: ARQ, NC and NCPC. In our simulation, the network parameters are set the same as that of a real network. We simulate a cellular network consisting of 7 cells, as shown in Fig. 4. The cell radius \( r_0 = 40(m) \); number of receivers in each cell is fixed at \( N = 15 \). The receivers are uniformly distributed over a cell. Other parameters such as carrier frequency, \( f = 2443(MHz) \); received power threshold, \( P_{th} = -80(dBm) \), receiver thermal noise \( N_0 = -130(dBm) \) and SIR = 3(dB). The Log-normal model parameters, \( \sigma = 3(dB) \) and \( \alpha = 3 \). Table I summarizes the simulation parameters. We first simulate a

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PARAMETERS OF SIMULATION CHANNEL.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>( f(MHz) )</td>
</tr>
<tr>
<td>10</td>
<td>2443</td>
</tr>
</tbody>
</table>

3Details of calculation can be found in [13]
versus number of transmission packets. As shown, ARQ and NCPC are correspondingly the worst and the best schemes. The network coding schemes gain compare with ARQ scheme because in the retransmission phase, error packets have been combined together before sending out. Furthermore, NCPC scheme can save more number of retransmissions and consumption energy since it considers combining error packets in a bigger radius cell. As seen in Fig. 5(a), power consumption of NC scheme is about 20% higher than that of NCPC scheme. The results for bandwidth efficiency are shown in Fig. 5(b). Again, NCPC and ARQ schemes have the worst and the best performances. BWEs of NCPC and NC are about 1.9 and 1.3 times better than that of ARQ, respectively. The improvement of NCPC over NC is about 44%. This is because in NCPC the center transmitter needs only one transmission instead of simultaneously several transmissions as in NC. By using this, NCPC scheme has eliminated the interference between transmitters.

Next, we investigate the network performance by adjusting the transmission power of the transmitter at the center cell. Fig. 6(a) shows that power consumptions of NC and ARQ schemes are constant while that of NCPC varies correspondingly with the change of ratio \( R = r_{23}/r_{0} \). Transmission power of the sender of the center cell increases correspondingly with the increasing of \( R \). Fig. 6(a) shows that power consumption of NCPC scheme is minimal at \( R = 1.23 \). This is because with a higher transmission power, the transmitter of the center cell can deliver information to not only the receivers of its cell but also the receivers of other neighbor cells. However, when increasing \( R > 1.23 \), power consumption of NCPC scheme increases. The reason is because when the center-cell sender increases its transmission power but the number of receivers with error packets inside the extension area do not increase accordingly. It has wasted the transmission energy. Therefore, \( R = 1.23 \) is considered as the optimal point of NCPC scheme in the simulation network scenario. Obviously, this optimal value is a function of cell radius, receiver condensability and channel characteristics.

Similarly, Fig. 6(b) presents the bandwidth efficiencies of different schemes. Again, bandwidth efficiencies of NC and ARQ schemes are constant while that of NCPC scheme increases and decreases correspondingly with the change of \( R \) around the optimal point.

VI. CONCLUSIONS

We have provided an analysis on network performance in terms of power consumption and bandwidth utilization of network coding techniques in a radio cellular network. Especially, a novel technique, joint network coding and adaptive power control, has been proposed to reduce the consumed power and bandwidth usage. The simulation results showed that our proposed technique can efficiently utilize in high saving energy and increasing throughput over those of the traditional techniques for a typical range of networks. The optimal value of relative ratio \( R \) is dependent on network topology, receiver condensibility and characteristics of transmission channel and how to obtain this value for a general network is still an interesting open problem.

REFERENCES