

Bitcoin Blockchain

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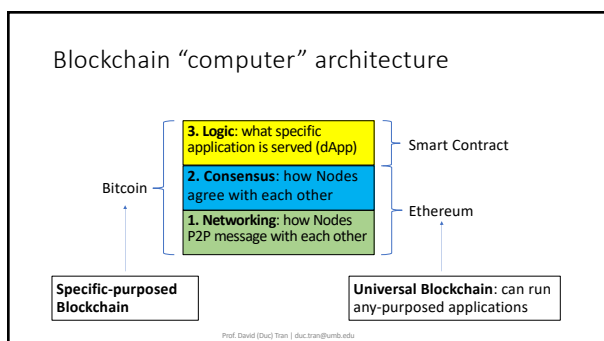
Bitcoin (Satoshi Nakamoto, 2008)

Bitcoin: A Peer-to-Peer Electronic Cash System

Satoshi Nakamoto
satoshi@gmx.com
www.bitcoin.org

Abstract. A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution. Digital signatures provide part of the solution, but the main

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Public vs. Private Blockchain?

- Public blockchain:** service available to everybody
- Private blockchain:** available to certain permitted participants

Case by case basis: can build "permissionless/permissioned" and "public/private"

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What is Bitcoin Blockchain?



- Bitcoin is a blockchain network implementing a **digital currency**
 - Keep money in your own (electronic) wallet
 - Transfer money P2P without intermediaries (e.g., banks)
 - No need for real-life identity
 - No double spending, no fake money possible
- Bitcoin currency:** a digital concept that represents "money" (a unit of value) in this blockchain network
- Open-source**
 - We can clone bitcoin software to build another blockchain network, **not necessarily about money**. Remember, it is a blockchain network first

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What is a valid transaction

A transaction = Alice sends "money" (coin) to Bob

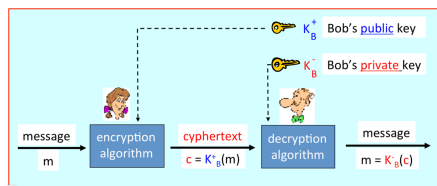
3 things to consider

- Bob is the **only recipient**, not anyone else
- Bob can verify that **Alice must undeniably be the sender**
- Alice has **enough money** to send

HOW?

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Public-Key cryptography



Guarantee Bob is the **only** recipient of the transaction

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Bitcoin Address

- To hold Bitcoin, need a **wallet** (like a bank account)
- Address of Bob is **hash value** of his **public key**

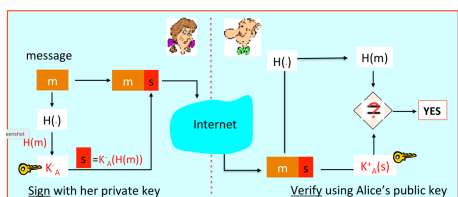
Address = HASH (public key)

Example: **1BvBMSEYstWetqTFn5Au4m4GFg7xJaNVN2**

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Private Key as a Digital Signature

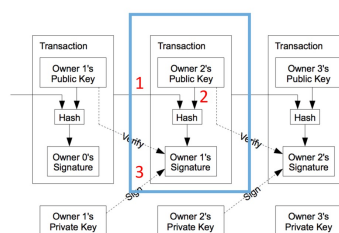


By signing, Alice proves that she is the sender; cannot deny that fact

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Sign and Verify a Transaction



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Ledger: Account-based

Create 25 coins and credit to Alice	ASSERTED BY MINERS
Transfer 17 coins from Alice to Bob	SIGNED(Alice)
Transfer 8 coins from Bob to Carol	SIGNED(Bob)
Transfer 5 coins from Carol to Alice	SIGNED(Carol)
Transfer 15 coins from Alice to David	SIGNED(Alice)

- **How to know if a transaction is valid?**
 – E.g., does Alice have the 15 coins to transfer to David?

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Ledger: Transaction-based

1	Inputs: \emptyset Outputs: 25.0→Alice	
2	Inputs: 1[0] Outputs: 17.0→Bob, 8.0→Alice	SIGNED(Alice)
3	Inputs: 2[0] Outputs: 8.0→Carol, 9.0→Bob	SIGNED(Bob)
4	Inputs: 2[1] Outputs: 6.0→David, 2.0→Alice	SIGNED(Alice)

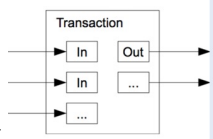
Verifying whether a transaction is valid is easy using "input" pointers

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A bitcoin transaction: enough fund to send?

Inputs

- Where the money comes from?
- Each input is the ID of an unspent transaction in which the sender receives money



Outputs

- Address of each recipient
- The amount of money sent

- 1) Sum of Outputs <= Sum of Inputs
- 2) Input transactions must be unspent yet

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Bitcoin Transaction Example

```

txid 90818a54288cc10d83f1abe9010d8ca87f6411a72b2e56a169f6c90219
{
  "hash": "90818a54288cc10d83f1abe9010d8ca87f6411a72b2e56a169f6c90219",
  "ver": 1,
  "type": "std",
  "lock_time": 0,
  "size": 226,
  "in": [
    {
      "txid": "1879f8795de44c308648d5dbabe10b1755524991232081e547b27939a",
      "vout": 0,
      "scriptSig": "304502210c1efcad5cdcd6df7c2a79d9e156523af7c229c78a7f1e4b8c300ab...[sig]"
    }
  ],
  "prev_outs": [
    {
      "value": "5.93100000",
      "scriptPubKey": "OP_DUP OP_HASH160 4b358739f7964b101279988ba0c0087ad OP_EQUALVERIFY OP_CHECKSIG"
    },
    {
      "value": "1678.04900000",
      "scriptPubKey": "OP_DUP OP_HASH160 5319e588cfc22a3f837cee934053460b339 OP_EQUALVERIFY OP_CHECKSIG"
    }
  ],
  "out": [
    {
      "value": "5.93100000",
      "scriptPubKey": "OP_DUP OP_HASH160 4b358739f7964b101279988ba0c0087ad OP_EQUALVERIFY OP_CHECKSIG"
    },
    {
      "value": "1678.04900000",
      "scriptPubKey": "OP_DUP OP_HASH160 5319e588cfc22a3f837cee934053460b339 OP_EQUALVERIFY OP_CHECKSIG"
    }
  ]
}
  
```

TX ID

INPUT TX ID

SIGNATURE

ADDRESS OF PAYEE

ADDRESS OF PAYEE

tx format version - currently at version 1
 in-counter - number of input amounts
 out-counter - number of output amounts
 tx_lock_time - should be 0 or in the past for the tx to be valid and included in a block
 size - of the transaction in bytes

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Transaction Lock Time

- Locktime defines the **earliest time that a transaction can be added to the blockchain.**
- It is set to **zero** in most transactions to indicate **immediate execution.**
- If locktime is **nonzero and below 500 million**, it is interpreted as a **block height**, meaning the transaction is not included in the blockchain prior to the specified block height.
- If it is **above 500 million**, it is interpreted as a **Unix Epoch timestamp** (seconds since Jan-1-1970).
- The use of locktime is equivalent to postdating a paper check.

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Transaction Outputs and Inputs (UTXO)

- Blockchain = a collection of **UTXO** transactions
 - Whenever a user receives bitcoin, that amount is recorded within the blockchain as a UTXO. Thus, a user's bitcoin might be scattered as UTXO amongst hundreds of transactions and hundreds of blocks
- UTXO are tracked by every full-node bitcoin client in a database held in memory, called the **UTXO pool**. New transactions consume (spend) one or more of these outputs from this pool
- There is **no "balance" or "account"** associated with a bitcoin address
 - The balance is computed by the wallet app: scanning the whole blockchain
- The bitcoin application can use several strategies to satisfy the purchase amount: combining smaller units, finding exact change, or using a single larger unit
 - Done by the user's wallet automatically and invisible to users

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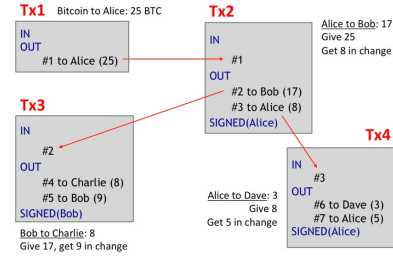
Python script to call blockchain.info API to find UTXO of an address

```

$ python get-utxo.py
ebadfaa92f1fd29e2fe296eda702c48bd11ff452313e986e99ddad9084062167:1 - 8000000 Satoshis
6596fd070679de9e405d52b51b81d644029108ec4cbe451454486796alecf:0 - 16050000 Satoshis
74d788804e2aae10891d72753d1520da1206e6f4f2048lcc1555b7f2cb44aca:0 - 5000000 Satoshis
b2affea89ff82557c60d635a2a3137b8f88f12ecce85082f7d0a1f82ee203aca:0 - 10000000 Satoshis
...
  
```

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Example



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Blockchain state

- State = The set of currently **unspent transactions** (UTXO: Unspent Transaction Output)

Old system state

UTXO A: value = 1 BTC, owner = Alice
UTXO B: value = 2 BTC, owner = Bob

Transaction

Input	Output
UTXO B	UTXO C (0.5 BTC, Bob) UTXO D (1.5 BTC, Alice)

↓

New system state

UTXO A: value = 1 BTC, owner = Alice
UTXO C: value = 0.5 BTC, owner = Bob
UTXO D: value = 1.5 BTC, owner = Alice

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Bitcoin Protocol

- Each node: upon receipt of a new transaction from application layer, send it to all nodes
- Other nodes: on receipt of a transaction from another node
 - Validate the transaction (make sure it is a valid transaction) → put in a mempool
 - Put valid transactions into **1 block**
 - **"Timestamp"** the block (to prove the birth of this block)
 - Add block to local blockchain copy
 - Broadcast this block to all other nodes
- Other nodes: on receipt of an arriving block (1st time)
 - Validate the block (Check the integrity of hash value with the previous block + check the validity of all transactions in the block)
 - Add this block to local blockchain copy

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Timestamping

- Satoshi: "I discovered Bitcoin in 2008"
- You: "I discovered Bitcoin in 2007"
- That means YOU are the inventor?
 - Yes, if **timestamp is correct!**
 - Need a **trusted timestamper**
- Timestamping** = a kind of cipher, to **prove the existence** of certain data (e.g., contracts, medical records, ...) before a certain time point against claims otherwise

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Bitcoin Timestamping

Timestamp = HASH(block + timestamp of prev_block)

Block

Item Item ...

Hash

→

Block

Item Item ...

- Timestamping** = evidence of **birth** of some information for the first time
- Each time a block is recorded on the blockchain, it **needs to be timestamped**
- Blockchain has no intermediaries, **which node will do the timestamping?**

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Timestamp (1610): Use Anagram

Galileo Galilei discovered the rings of Saturn in 1610. He wrote **smaismrmilmepoetaleumibunenugttauiras** to claim **alNssimum planetam tergeminum observavi** ("I observed the most distant planet to have a triple form")

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Robert Hooke (1660)

When **Robert Hooke** discovered Hooke's law in 1660, he wanted to claim invenNon without showing content because he was not ready to publish; so he wrote

"ceiinossttuv"

(anagram of "**ut tensio, sic vis**" with Latin meaning "**as the tension, so the force**")

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Anagram: Các ví dụ vui

- "resoul" = "fluster"
- "funeral" = "real fun"
- "adultery" = "true lady"
- "customers" = "store scum"
- "forty five" = "over fifty"
- "William Shakespeare" = "I am a weakish speller"

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Modern Timestamping

The diagram illustrates the trusted timestamping process. It is divided into two main sections: 'Within a company' and 'Timestamping Authority (TSA)'.
 1. **Within a company:** A document labeled 'Data' is processed. A 'Calculate hash' step produces a binary hash '1011...1010'. This hash is then combined with a 'Timestamp' to form 'Data + [0010...0101] + Timestamp'.
 2. **Timestamping Authority (TSA):** The company sends the hash '1011...1010' to the TSA. The TSA calculates its own hash '0010...01011'. The TSA then applies its private key to this hash to create a digital signature. The signed timestamp and hash are returned to the requester.
 3. **Final Step:** The requester combines the original data with the received signed timestamp and hash: 'Data + [0010...01011] + Timestamp'. The final result is 'Store together'.

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Checking Timestamp

The diagram shows how to check a timestamp. It starts with 'Data + Timestamp + [0010...0101]'.
 1. **Calculate hash:** The data is hashed to produce '1011...1010'.
 2. **Calculate hash:** The timestamp is hashed to produce '0010...01011'.
 3. **Apply public key of Timestamping Authority (TSA):** The TSA's public key is used to verify the signature on the timestamp.
 4. **Comparison:** The two hashes are compared. If they match, the timestamp is valid. If not, it is invalid.
 Text below: "If data or timestamp are changed, hashcode will disagree".
Source: wikipedia
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Bitcoin Timestamping

- Anyone can timestamp → called a **Miner** (mining bitcoin)
- Only need a computer running the **Bitcoin Client software**
- **Incentivized to participate** → receive **6.25 BTC** (currently) for each good block created

Challenge:

- Everybody wants to create the next block, **choose whom?** (**Consensus Problem**)
- How to **discourage bad nodes?**

Solution: **Proof-of-Work**

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"Proof of Work" (PoW)

Miner Node

- Solve 1 **difficult** "computational puzzle"
- If solving first → successful block → timestamp done!

Why PoW?

- **It takes time to solve** → discourage bad nodes from abusing
- **Slow to create a block** → scarcity for bitcoin → stable/increased price
- Stronger computer → faster timestamping → **good competition**

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POW is not new

- Proposed by Dwork & Naor to prevent email spamming (1992)
 - Every time you send an email, your computer must solve a computational puzzle
 - The recipient's email program ignores your email if you don't attach the solution to the puzzle.
- A similar idea was proposed in **HashCash** by Adam Back (1997) for anti-denial-of-service
 - Bitcoin extends the PoW idea of HashCash

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PoW Problem for Bitcoin

- Problem:** Find **Nonce (32-bit)** such that $HASH(\text{block}) < 2^d$
 - Difficulty level (d) is dynamically adjusted such that only 1 block can be added in every 10 minutes
 - To solve, the **only way** is to try **Nonce = 0, 1, 2, ...** until the condition above is met

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Mining Pseudocode

```

TARGET = (65535 << 208) / DIFFICULTY;
coinbase_nonce = 0;
while (1) {
  header = makeBlockHeader(transactions, coinbase_nonce);
  for (header_nonce = 0; header_nonce < 232; header_nonce++) {
    hash_value = SHA256(SHA256(makeBlock(header, header_nonce)));
    if (hash_value < TARGET) break; //block found!
  }
  coinbase_nonce++;
}
    
```

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What If No NONCE Value is Found?

- The new block always contains a **coinbase transaction** (to claim reward)
- There is a "coinbase" field where you can enter arbitrarily
- If no NONCE is found to satisfy the zero prefix, try a different coinbase value and repeat PoW

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Mining Difficulty

- Increased after every 2016 blocks (2 weeks)

- Each miner independently computes the difficulty and will only accept blocks that meet the difficulty that they computed

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Upgrade Hardware to "mine" faster

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Decentralized Consensus

- In Bitcoin, each transaction is broadcast to the network
 - each node has its own pool of pending transactions
- Challenge:** how to make sure that a pending transaction is inserted only **ONCE** to the blockchain?
- Need a **decentralized consensus protocol!**

N nodes each provide a value. Some nodes are **faulty** or malicious. A distributed consensus protocol **must**

- Must terminate with a value agreed by all the honest nodes
- This value must have been generated by an honest node

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How to know which blockchain copy is good?

- Each node has its own **local copy** the blockchain, updated at different times independently
- Nodes may be **dishonest** (sending bad blocks to other nodes, do not process good blocks, etc.)

Consensus

Always treat **the longest copy as the correct one**

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Bitcoin (Nakamoto) Consensus Protocol

- A **breakthrough** invention!
- **Randomized + Asynchronous**: tolerate $f < n/2$ corruptions
- **First** to reach consensus in **large-scale, permissionless** environments
 - Nodes are free to join at any time
 - No a priori knowledge of the identities of the nodes → participants must communicate through **unauthenticated** channels
- In contrast, classic consensus is **small-scale** and **permissioned**
 - Only a preconfigured, known set of nodes can join the protocol

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Nakamoto Protocol: Proof of Work

- “Permissionless” is **difficult** because of “Sybil attack”
 - Due to **unauthenticated** communication channels, a player can **impersonate** many others to **outnumber** the honest players and disrupt the consensus
- **Proof of Work (PoW)**: To discourage Sybil attacks, participants have to “pay” a cost to join the protocol
 - By having to solve a computationally-expensive puzzle to cast votes
 - A player’s voting power is proportional to its computational power
 - PoW guarantees **consistency** and **liveness** as long as **>50%** is honest

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Mining

- **Block structure**: $\mathbf{b} = (\mathbf{h}_{\text{prev}}, \text{pow}, \text{transactions}, \mathbf{h})$
 - **h_{prev}**: hash of the previous block
 - **pow**: an unknown number (called “proof of work”) to be found
- **Mining**: to create a block \mathbf{b}
 - Find **pow** and set **h** accordingly such that

$$\mathbf{h} = \text{Hash}(\mathbf{h}_{\text{prev}}, \text{pow}, \text{transaction}) < \text{difficulty_threshold}$$

(**difficulty_value** is chosen such that **only 1 block** is created per **10 minutes**)

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Broadcast and Update

- **The mining node**: After mining a block
 - **Add** block to local chain
 - **Broadcast** local chain to all other nodes
- **The other nodes**: when hearing a **valid chain**
 - **Valid** = iff each block is consistent with the hash of the previous block
 - **Replace** the local chain with the received chain **if the latter is longer**
 - “**Finalized**” chain = this local chain up to the **K** last block (e.g., $K=6$ enough)

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Mining Incentive in Bitcoin

- **Classic** consensus (google, facebook): focus on fault tolerance, no incentive because the components belong to the institution
 - **Blockchain**: decentralized, permissionless
 - PoW: discourage bad players
 - Incentive: encourage miners to create good blocks
- Per-block mining reward**
- **Block reward**: initially, block reward is 50 bitcoins. After every 210,000 blocks mined (~4 years), the reward is halved; eventually becoming zero by year 2140 (when all 21 million bitcoins are minted)
 - **Transaction fee**: every transaction can specify a fee to pay to the miner that includes the transaction (higher fees speed up transaction processing)

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Mining Difficulty

- Choose **difficulty threshold** = $p2^m$ (where m = bit-length of hash)
- Where p = **prob** {a node mines a block in a round}
- Prob {a **good** block is mined in a round} = $1 - (1 - p)^{0.51n}$
- #**rounds** to mine a new **good** block = $1 / (1 - (1 - p)^{0.51n}) \approx 1 / (0.51pn)$
- It takes Δ rounds to propagate this block to all honest nodes
- The block mining **efficiency** ratio can be

$$\frac{\frac{1}{0.51pn}}{\frac{1}{0.51pn} + \Delta} = \frac{1}{1 + 0.51pn\Delta} \approx 1 - 0.51pn\Delta$$

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Choosing Mining Difficulty

- q : **fraction of dishonest** mining power (hash rate)
- To be secure, honest hash rate must be higher than the dishonest

$$(1-q)(1-0.51pn\Delta) > (1+\phi)q \quad (\text{here, } \phi \text{ chosen arbitrary small})$$

$$0.51pn\Delta < 1 - (1+\phi)q/(1-q)$$

$$p < (1 - (1+\phi)q/(1-q))/(0.51n\Delta)$$

- The **smaller** $p \rightarrow$ the **more difficult** mining
- In practice, **choose** $p < \min(0.5/(2n\Delta), (1 - (1+\phi)q/(1-q))/(2n\Delta))$
- The **larger** network delay Δ , the **weaker** security

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Consistency Guarantee

- $C1$ = some honest node's longest chain (last K blocks removed) in round r
- $C2$ = some honest node's longest chain (last K blocks removed) in round $t \geq r$

Consistency: $C1$ must be a **prefix** of $C2$

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Chain Growth Guarantee

- #honest nodes that mine a block in each round = $(1-q)np$
- #good blocks added $> (1-q)np(1 - 2n\Delta)$

After any $t \geq K/(qnp)$ rounds, any honest node's chain will have added at least $(1-q)np(1 - 2n\Delta)t$ blocks

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Liveness Guarantee

In every window of consecutive K blocks in honest nodes' longest chains, more than $\mu := 1 - 1/(1+\phi)$ fraction are mined by honest nodes

What that means

- Every now and then, an honest block is added to the blockchain
- Hence, **liveness**: transactions submitted will be recorded in honest nodes' finalized chains fairly soon (after $\Theta(K/((1-q)np) + \Delta)$ rounds)

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Mining Fairness: It is not fair!

- Fairness** iff the honest block creation rate = the honest hash rate
- the **honest block rate** is $\mu := 1 - 1/(1+\phi)$
- the **honest hash rate** is $1-q$
- Assume $\Delta=0$ (ideal case for honest nodes) and set $(1-q) = (1+\phi)q$
- We have $\mu := 1 - 1/(1+\phi) = 1-q/(1-q) < (1-q)$ (not fair!)
- Attack: a coalition with $q=49\%$ hash rate can control 96% the blocks!

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Selfish Mining Attack

- Capitalize on the unfairness of Nakamoto consensus: damage transactions, earn block mining rewards
- Selfish miner**: mine a block B, but **withhold** it until some honest miner also mines a block B' at the same length (block number) as B
 - When this happens, immediately releases B
 - If B is **transmitted faster** than B', honest nodes' work (B') is **wasted**
 - Feasible**: bad nodes collude with the network relay to deliver blocks to miners

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Selfish Mining Attack

Majority is not Enough: Bitcoin Mining is Vulnerable*

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Abstract. The Bitcoin cryptocurrency records its transactions in a public log called the blockchain. Its security rests critically on the distributed protocol that maintains the blockchain, run by participants called miners. Conventional wisdom asserts that the mining protocol is incentive-compatible and secure against colluding minority groups, that is, it incentivizes miners to follow the protocol as prescribed. We show that the Bitcoin mining protocol is not incentive-compatible. We present an attack with which colluding miners obtain a revenue larger than their fair share. This attack can have significant consequences for Bitcoin: Rational miners will prefer to join the selfish miners, and the colluding group will increase in size until it becomes a majority. At this point, the Bitcoin system ceases to be a decentralized currency. Unless certain assumptions are made, selfish mining may be feasible for any group size of colluding miners. We propose a practical modification to the Bitcoin protocol that protects Bitcoin in the general case. It prohibits selfish mining by pools that controlled less than 1/4 of the resources. This threshold is lower than the wrongly assumed 1/2 bound, but better than the current reality where a group of any size can compromise the system.

1 Introduction

Bitcoin [24] is a cryptocurrency that has recently emerged as a popular medium of exchange, with a rich and extensive ecosystem. The Bitcoin network runs at

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The Double Spending Problem

- A malicious miner makes a payment, then secretly creates a second conflicting transaction in a new block, which allows him to recover the funds
- Feasible if he controls **q>50%** hashrate → mining **faster** than the rest of the network → make his local chain **the longest**
- But due to randomness, if **q<50%**, there is **still a non-zero chance**
- How to minimize risk?** When somebody pays you, **wait some time** before delivering service. In bitcoin, wait for **6 block confirmations**

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Why 6 Block Confirmations?

- Consider a miner with a fraction $0 < p \leq 1$ of the total hash rate
- The whole network takes on average $\tau_0 = 10$ minutes to create a block
- The miner takes on average $t_0 = \tau p$ time to create a block
- $T = T_1, T_2, \dots, T_n$: inter-block mining time of block 1, block 2, ...
- Because mining is a Markov process (i.e., memoryless), T follows an **exponential distribution**

$$f_i(t) = \alpha e^{-\alpha t}$$

where $\alpha = 1/t_0$

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Poisson Law

The time needed to discover n blocks is $S_n = T_1 + T_2 + \dots + T_n$

$N(t)$: #blocks validated at time t is **Poisson** with mean value αt

$$S_n = T_1 + T_2 + \dots + T_n$$

The random variable S_n follows the n -convolution of the exponential distribution and, as is well known, this gives a Gamma distribution with parameters (n, α) ,

$$f_{S_n}(t) = \frac{\alpha^n}{(n-1)!} t^{n-1} e^{-\alpha t}$$

with cumulative distribution

$$F_{S_n}(t) = \int_0^t f_{S_n}(u) du = 1 - e^{-\alpha t} \sum_{k=0}^{n-1} \frac{(\alpha t)^k}{k!}$$

From this we conclude that if $N(t)$ is the process counting the number of blocks validated at time $t > 0$, $N(t) = \max\{n \geq 0; S_n < t\}$, then we have

$$\mathbb{P}[N(t) = n] = F_{S_n}(t) - F_{S_{n+1}}(t) = \frac{(\alpha t)^n}{n!} e^{-\alpha t}$$

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Winning the race against the malicious

- q : hash rate of the malicious
- $N^*(t)$ = #malicious blocks at time t , which is Poisson
- $X_n = N^*(S_n)$: #malicious blocks for every n consecutive honest blocks
 - S_n = time at which the honest has mined n blocks
 - X_n = a **negative binomial variable** with parameters (n, p)

$$\mathbb{P}[X_n = k] = p^k q^n \binom{k+n-1}{k}$$

How likely the malicious wins if behind the dishonest by z blocks? (similar to the classical gambler's ruin problem)

$$P(z) = I_{qq}(z, 1/2)$$

where $I_x(a, b)$ is the incomplete regularized beta function

$$I_x(a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1} (1-t)^{b-1} dt$$

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Probability to win the race

$\lambda = z \frac{q}{p}$

p = probability an honest node finds the next block
 q = probability the attacker finds the next block

- The bad node and good node are racing to grow their blockchain to be the longer. Unless $q > 50\%$, the bad node will win with probability

• Prob (attacker catch up) $\rightarrow 0$ quickly as z large

$$\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} \begin{cases} (q/p)^{z-k} & \text{if } k \leq z \\ 1 & \text{if } k > z \end{cases} = 1 - \sum_{k=0}^{z-1} \frac{\lambda^k e^{-\lambda}}{k!} (1 - (q/p)^{z-k})$$

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Bitcoin Nodes

GLOBAL BITCOIN NODES DISTRIBUTION
 Reachable nodes as of Sat Apr 17 2021 16:47:26 GMT+0700 (Indochina Time).
9796 NODES
 24-hour charts >

Top 10 countries with their respective number of reachable nodes are as follow.

RANK	COUNTRY	NODES
1	United States	1879 (19.18%)
2	n/a	1807 (18.45%)
3	Germany	1795 (18.33%)
4	France	616 (6.29%)
5	Netherlands	417 (4.26%)
6	Canada	321 (3.28%)
7	United Kingdom	290 (2.96%)
8	Russian Federation	260 (2.65%)
9	China	203 (2.07%)
10	Singapore	157 (1.60%)

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Bitcoin is slow

- Block size = ~ 1MB
- Transaction size = ~250 bytes
- Each block = ~4,000 transactions
- Block rate = 1 per 10 minutes
- Transaction rate = 7 transactions per second**
 - Particularly slow for applications that support real-time transactions
 - Visa: 2000-10,000 transactions/s
 - Paypal: 100 transactions/s

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Blockchain storage size

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Lightweight Nodes

- Vast majority of nodes are **lightweight nodes**
 - Run the thin or Simple Payment Verification (SPV) client software
 - Don't store the full version of the blockchain
- For example, If you use a wallet program, it would typically incorporate an SPV node. Only store the block headers and transactions that represent payments to your addresses.

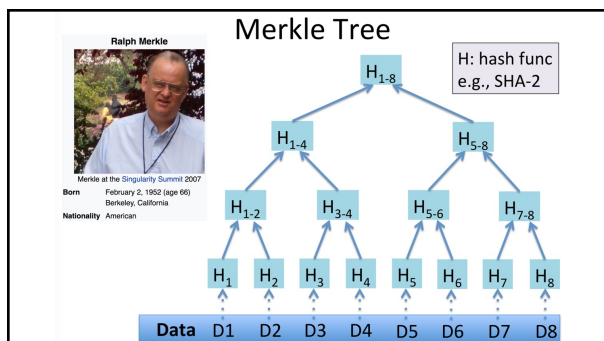
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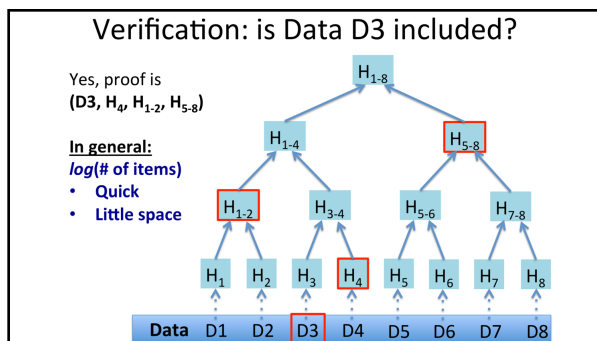
Too Much Disk Space?

- Recall, a coin = a chain of transactions
- Once the latest transaction is buried under enough blocks, we can **discard the previous spent transactions** to save disk space
- But would that break the block's hash?
- Solution**
 - Create the block's hash from its transactions based on the **Merkle Tree**

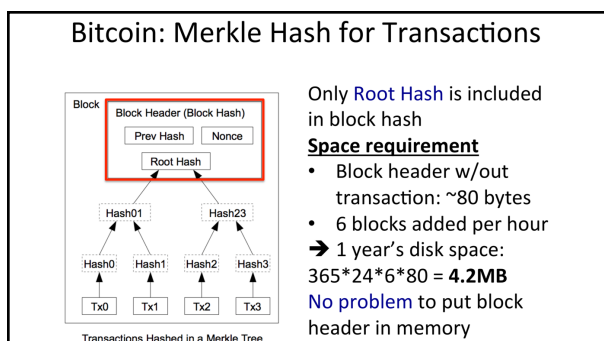
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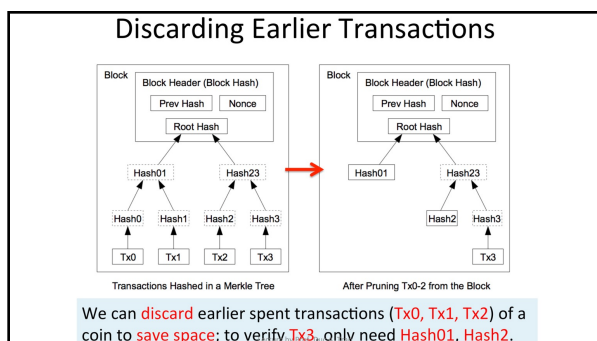
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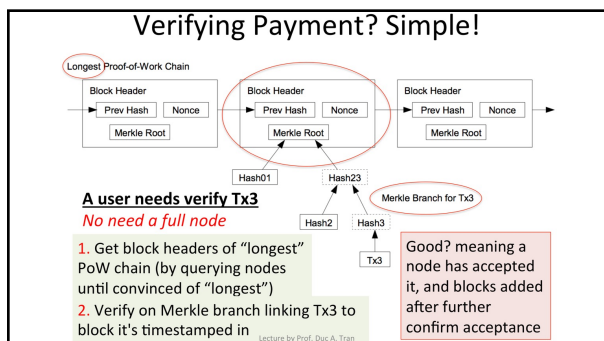
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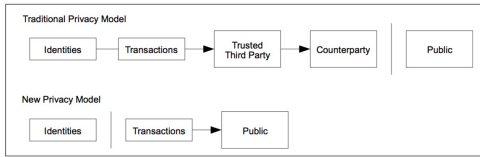
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Failed Verification?

- Possible, when network is overpowered by an attacker who fabricates transactions
 - Need to **alert** network nodes
- When a node is alerted
 - Download the full block and alerted transactions to confirm the inconsistency
- Businesses that receive frequent payments should run their own FULL nodes for more independent security and quicker verification.

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Privacy



Total privacy: Only transactions can be seen by the public. Addresses are visible but nobody knows WHO owns them.

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Before Bitcoin (<2008)

Similar solutions, or relevant:

- **Hashcash** (Adam Back, 1997): introduced "Proof of Work" (PoW)
- **bMoney** (Wei Dai, 1998): PoW + Peer-to-Peer (similar to Bitcoin)
- **Bitgold** (Nick Szabo, 1998)
- **Reusable PoW** (Hal Finney, before 2008)

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