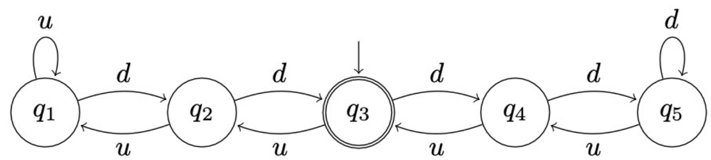
CS420 Homework 1 Solutions

1.3

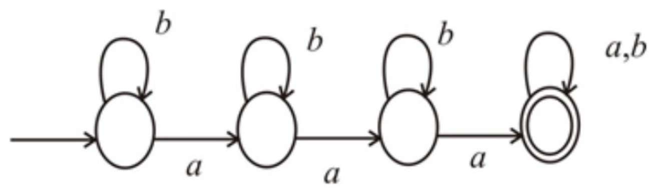
The state diagram of the machine will be as follows



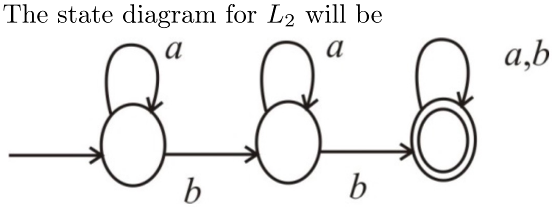
1.4.a

Let $L = \{w|w$ has at least three a's and two b's$\}$, then $L_1 = \{w|w$ has at least three a's$\}$ and $L_2 = \{w|w$ has at least two b's$\}$

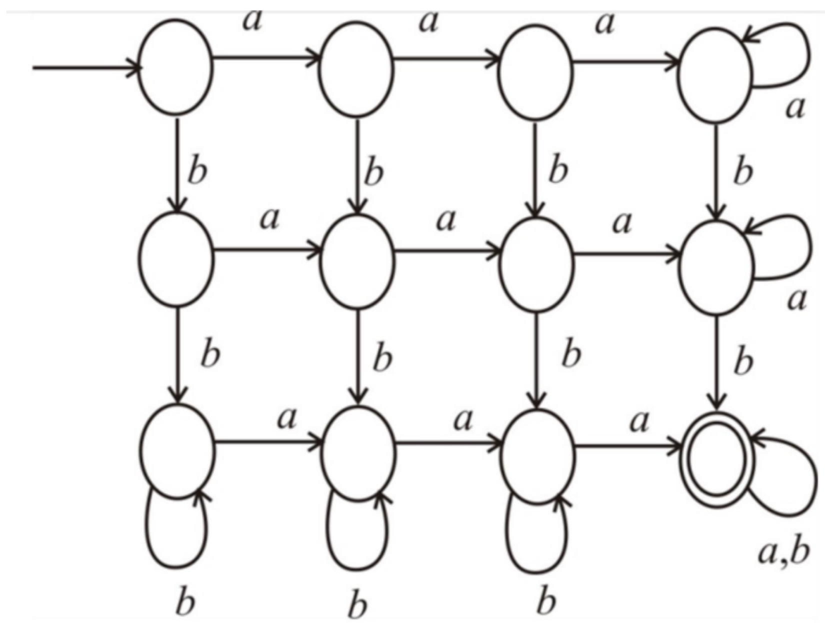
The state diagram for L_1 will be



The state diagram for L_2 will be



The DFA M which is the intersection of the languages L_1 and L_2 will have $Q = Q_1$ x $Q_2 = 12$ states and one accept state. The state diagram for the DFA M will be as follows.

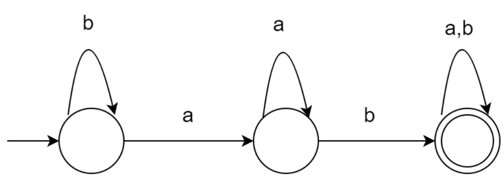


1.5.c

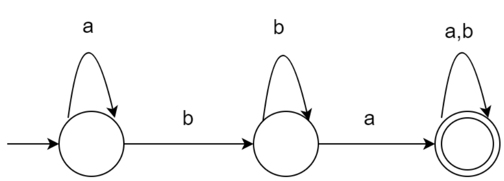
We have L= {w|w contains neither the substrings *ab* nor *ba*}

then $\bar{L}$ = {w|w contains either the substrings *ab* or *ba*}.

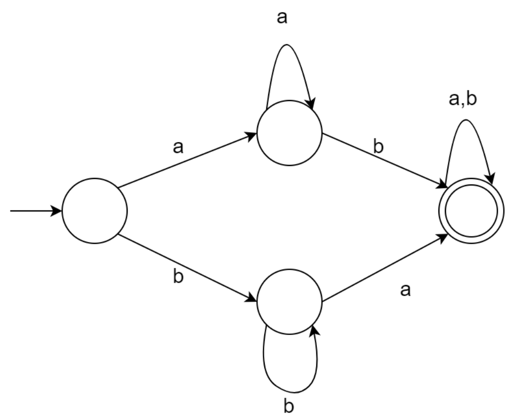
Firstly, we design the state diagram of DFA that accepts w contains *ab*



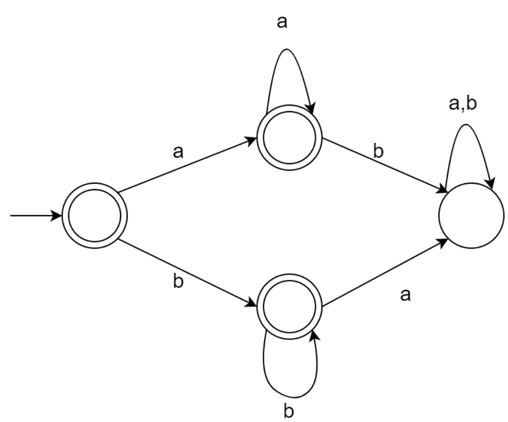
And similarly, the state diagram of DFA that accepts w contains *ba*



Hence, the state diagram of DFA that accepts $\bar{L}$ is the combination of two diagrams above, as follows

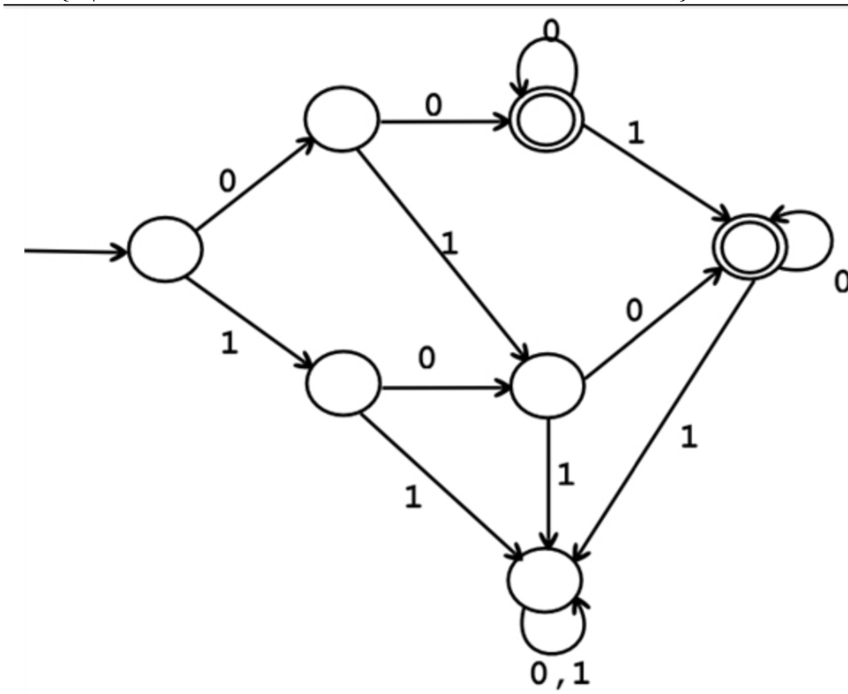


To get the state diagram of the DFA that accepts L we need to replace the accepting states F with $Q \setminus F$



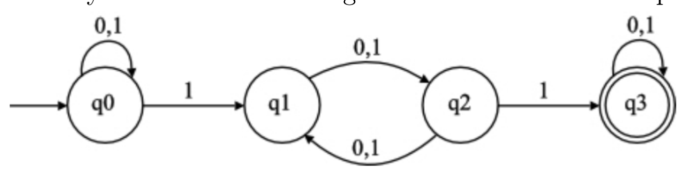
1.6.j

The state diagram of the DFA which accepts the language
$L = \{w|w$ contains at least two 0's and and at most one 1$\}$ is



1.13

The language L is a set of strings that do not contain a pair of 1's that are separated by an odd number of symbols, then the language $\bar{L}$ would be the set of strings that contain atleast one pair of 1's that are separated by an odd number of symbols. The state diagram of the NFA that accepts $\bar{L}$ would be



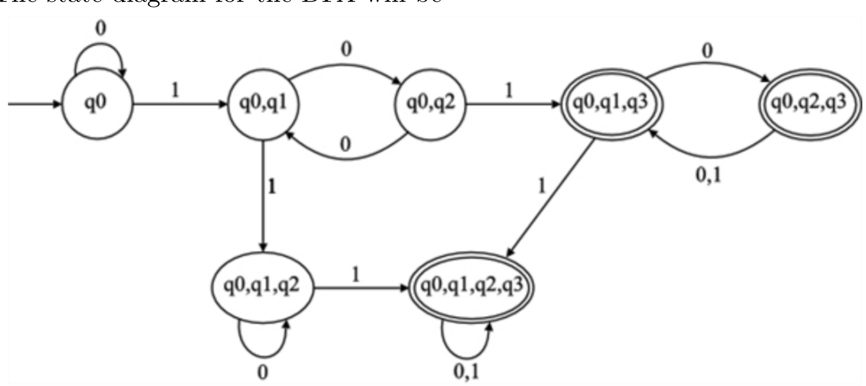
The transition table for the above is

State	0	1
q_0	q_0	$\{q_0, q_1\}$
q_1	q_2	q_2
q_2	q_1	$\{q_1, q_3\}$
q_3	q_3	q_3

We now need to convert the NFA to a DFA, which will have the following transition table

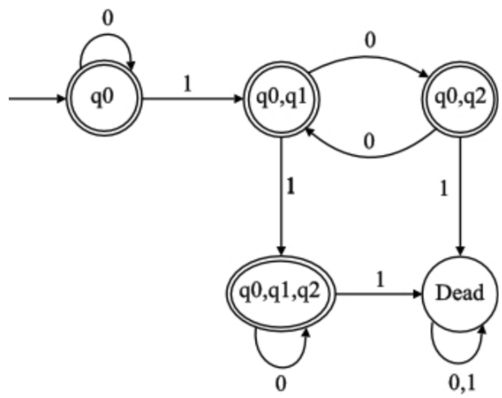
State	0	1
q_0	q_0	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_3\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$
$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_3\}$

The state diagram for the DFA will be



To get the DFA for the language L, we need to compliment the above DFA, we can also simplify the state diagram converting the states $\{q_0, q_1, q_2, q_3\}$,$\{q_0, q_1, q_3\}$

and $\{q_0, q_2, q_3\}$ into one state.

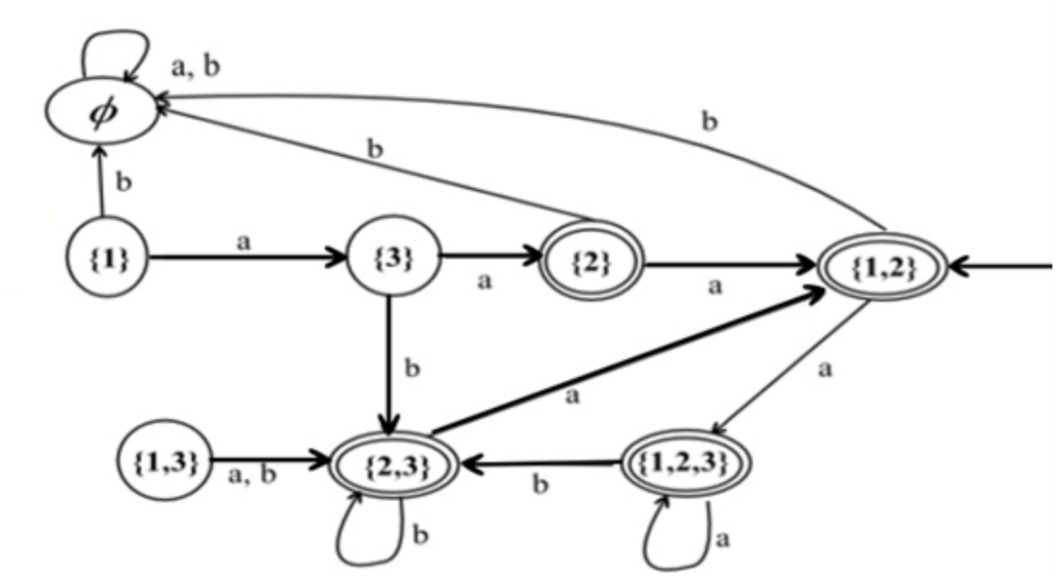


1.16.b

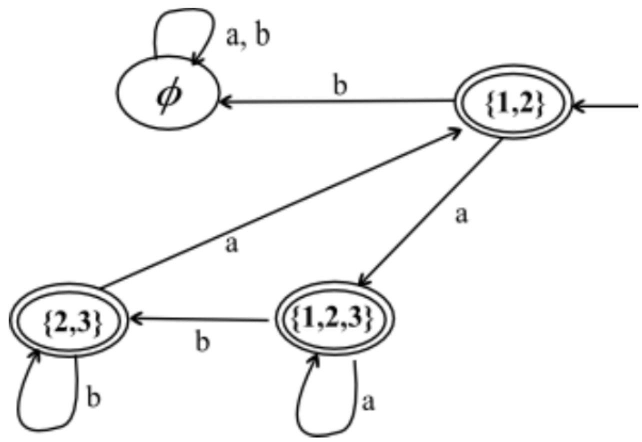
Let $N = (Q, \sum, \delta, q_0, F)$ be the definition the the NFA. Let $D = (Q', \sum, \delta', q_0', F')$ be the DFA that is equivalent to N. Using the theorem 1.9, we have

$Q' = \mathcal{P}(Q) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$. Now we need to calculate δ'

$\delta'(\phi, a) = \delta'(\phi, b) = \phi$
$\delta'(\{1\}, a) = E(\delta(1, a)) = \{3\}$
$\delta'(\{1\}, b) = E(\delta(1, b)) = \phi$
$\delta'(\{2\}, a) = E(\delta(2, a)) = \{1, 2\}$
$\delta'(\{2\}, b) = E(\delta(2, b)) = \phi$
$\delta'(\{3\}, a) = E(\delta(3, a)) = \{2\}$
$\delta'(\{3\}, b) = E(\delta(3, b)) = \{2, 3\}$
$\delta'(\{1, 2\}, a) = E(\delta(1, a)) \cup E(\delta(2, a)) = \{1, 2, 3\}$
$\delta'(\{1, 2\}, b) = E(\delta(1, b)) \cup E(\delta(2, b)) = \phi$
$\delta'(\{1, 3\}, a) = E(\delta(1, a)) \cup E(\delta(3, a)) = \{2, 3\}$
$\delta'(\{1, 3\}, b) = E(\delta(1, b)) \cup E(\delta(3, b)) = \{2, 3\}$
$\delta'(\{2, 3\}, a) = E(\delta(2, a)) \cup E(\delta(3, a)) = \{1, 2\}$
$\delta'(\{2, 3\}, b) = E(\delta(2, b)) \cup E(\delta(3, b)) = \{2, 3\}$
$\delta'(\{1, 2, 3\}, a) = E(\delta(1, a)) \cup E(\delta(2, a)) \cup E(\delta(3, a)) = \{1, 2, 3\}$
$\delta'(\{1, 2, 3\}, b) = E(\delta(1, b)) \cup E(\delta(2, b)) \cup E(\delta(3, b)) = \{2, 3\}$
Now $q_0' = E(q_0) = E(1) = \{1, 2\}$
$F' = \{R \in Q' | R \text{ contains an accept state of } N\} = \{\{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$
Drawing the state diagram using the DFA above we get,

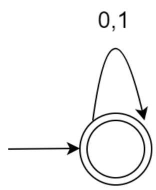


We can simplify the diagram by removing states that are not reachable from the start state

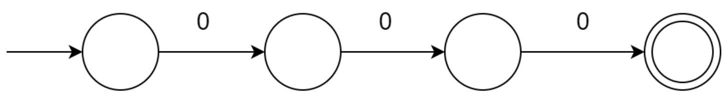


1.19.a

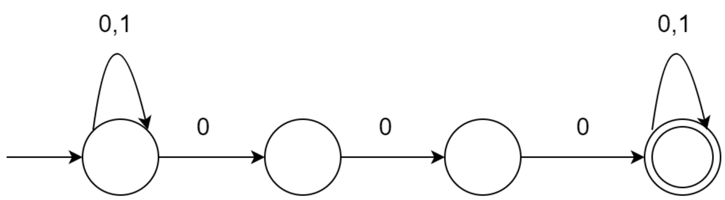
We can construct NFA for the regular expression $(0 \cup 1)^*000(0 \cup 1)^*$ as follows the state diagram A for $(0 \cup 1)^*$:



the state diagram B for 000:

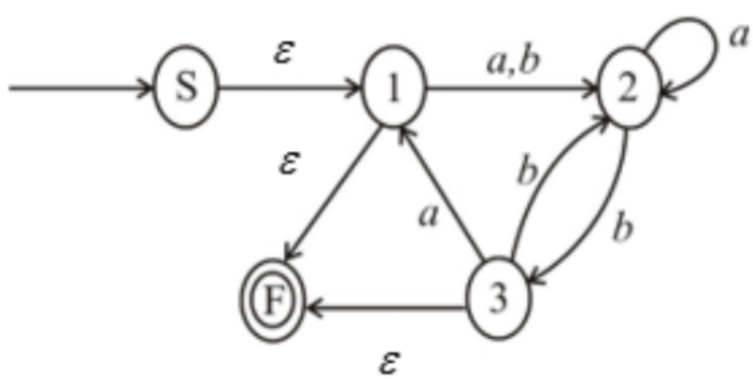


Then, we combine them: ABA:

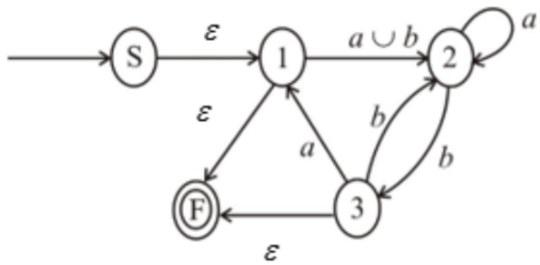


1.21.b

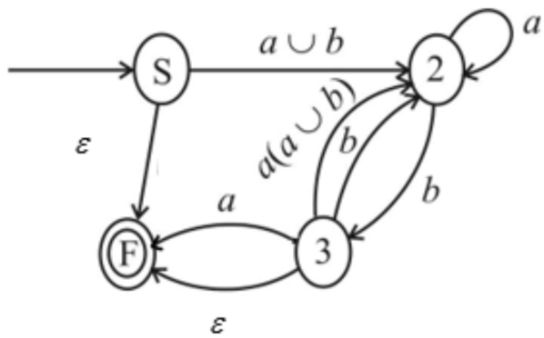
First we convert the DFA to a Generalized DFA by adding a new start and



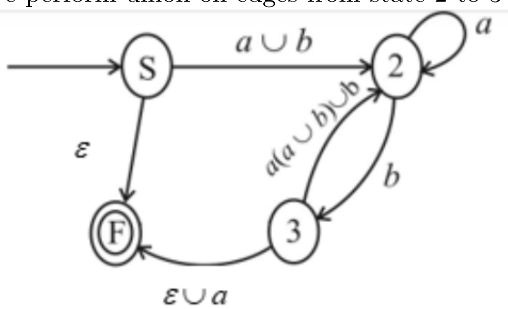
accept state.
We perform union on the edge from state 1 to 2



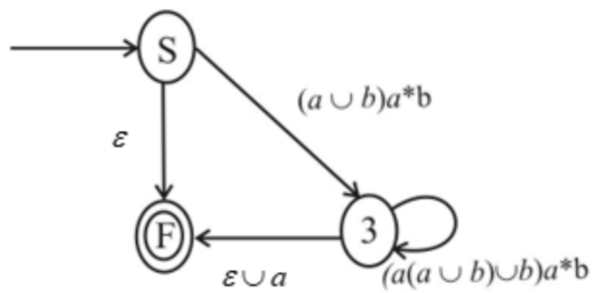
We eliminate state 1 as follows



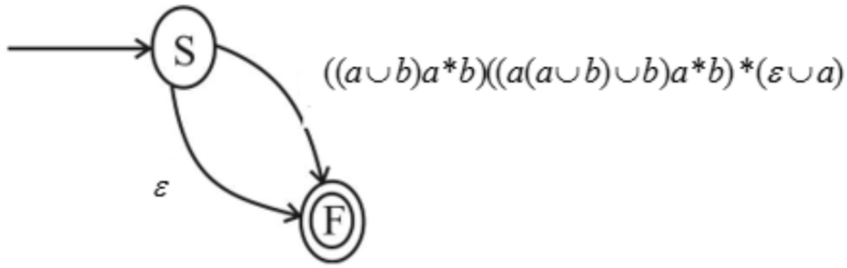
We perform union on edges from state 2 to 3 and from 3 to final state



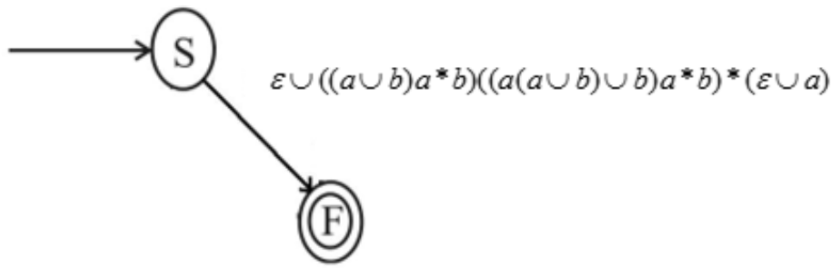
We then eliminate state 2 as follows



We eliminate state 3 next



We perfom union on edge from S to F to get



The regular expression of the DFA is
$\varepsilon \cup ((a \cup b)\, a^* b)\, ((a\, (a \cup b) \cup b)\, a^* b)^* \, (\varepsilon \cup a)$