1.46 c

$L = \{ w \mid w \in \{0,1\}^* \text{ is not palindrome} \}$.

We will prove by contradiction. Let's assume that L is regular, then

$\overline{L} = \{ w \mid w \text{ is palindrome} \}$ is also regular.

and p is it's pumping length.

Let's consider string $s = 0^p 1 0^p$, $|s| = 2p+1 > p$

Then, following pumping lemma, s can split to 3 parts

$s = xyz$, such that:

1/ $|xy| \leq p$ then y only includes 0s

2/ $|y| > 0$

Let's pump it down, $s' = xy^0z$.

Thus, s' has form $0^k 1 0^p$ with $k < p$ ($|y| > 0$)

$\Rightarrow s' \notin \overline{L}$ (contradiction)

Therefore, L is not palindrome

1.46.d.

$L = \{ wtw \mid w, t \in \{0,1\}^* \}$

Let's assume that L is regular and it's pumping length is p.

Let's consider string $s = 0^p 1 1 0^p 1$, $|s| = 2p+3 > p$.

Thus, s can be splitted to 3 parts: $s = xyz$ such that

1, $k = |y| > 0$

2, $|xy| \leq p$, then y consists ony 0s

3, $\forall i$, $xy^i z \in L$

Choose $i = 2$, we have $s' = xy^2 z = 0^{p+k} 1 1 0^p 1$, $k > 0$

This string can't have form wtw since:

last symbol is 1, w must have 1 at the end.

Thus length of w is at least $p+k+1 > p+1$, this make w consists at least 2 symbol 1s. However, there are only 3 1s, can't divided to 2 w. (Contradiction)

Therefore, L is not regular.

2.4.c

$L = \{ w \mid$ the length of w is odd $\}$.

$S \to 0 \mid 1 \mid AA0 \mid AA1$

$A \to 0 \mid 1$

2.28.b

$L = \{ w \mid$ the number of a's $=$ b's $\}$

$S \to aSbS \mid bSaS \mid \varepsilon$

2.28c

$L = \{ w \mid \#a \geq \#b \}$

$S \to aSbS \mid bSaS \mid aS \mid \varepsilon$

2.30.a

$L = \{ 0^n 1^n 0^n 1^n, n \geq 0 \}$

Let's assume that L is context-free, with pumping length p

Let's consider string $s = 0^p 1^p 0^p 1^p$, $|s| = 4p > p$.

Then, follow pumping lemma, we can split s into 5 parts

$s = uvxyz$, such that

1/ $|vy| > 0$
2/ $|vxy| \leq p$ so 'vxy' has form 0^a1^b or 1^b0^a $(a, b \geq 0)$

3, $\forall i$, $s' = uv^ixy^iz \in L$.

With $i = 0$, $s' = uxz$. After pumping down, at least 1 area 0's or 1's has length p and at least 1 area 0's or 1's has length less than p
Thus $s' \notin L$ (contradiction)
Therefore L is not context free

2.31

$B = \{w \mid w$ is palindrome and w contain an equal number of 0s and 1s $\}$

Let's assume B is context free with pumping length p.

Let's consider string $s = 0^p1^{2p}0^p$, follow pumping lemma, s can be splitted into 5 parts

$s = uvxyz$, ST:

1, $k = |vy| > 0$

2, $|vxy| \leq p$

3, $\forall i \geq 0$, $s' = uv^i xy^i z \in B$.

If $i = 0$, $s' = uxz$

⊕ v and y include only 1 symbol 0s or 1s.

- 0s : $s' = 0^{p-k} 1^{2p} 0^p \notin L$ $(k > 0)$
- 1s : $s' = 0^p 1^{2p-k} 0^p \notin L$ $(k > 0)$

⊕ v or y include both 0s and 1s

- $0^a 1^b$: $s' = 0^{p-a} 1^{2p-b} 0^b \notin L$
- $1^a 0^b$: $s' = 0^p 1^{2p-a} 0^{p-b} \notin L$

=> Contradiction

Therefore, B is not context free

2.32

Let's assume that C is context free with pumping length p.

Let's consider string $s = 1^p 3^p 2^p 4^p$, $|s| = 4p > p$

Then s can split into 5 part $s = uvxyz$, ST.

1, $k = |vy| > 0$

2, $|vxy| \leq p$ then vxy only consists no more than 2 alphabet symbol.

3, $\forall i \geq 0$, $s' = uv^i x y^i z \in C$

With $i = 0$, $s' = uxz$, since $|vy| > 0$ and vxy only consists no more than 2 continuos alphabet symbol, the number of 1s is not equal to the number of 2's or the number of 3s is not equal to the number of 4s then $s' \notin C$ (contradiction)

Therefore C is not context free.