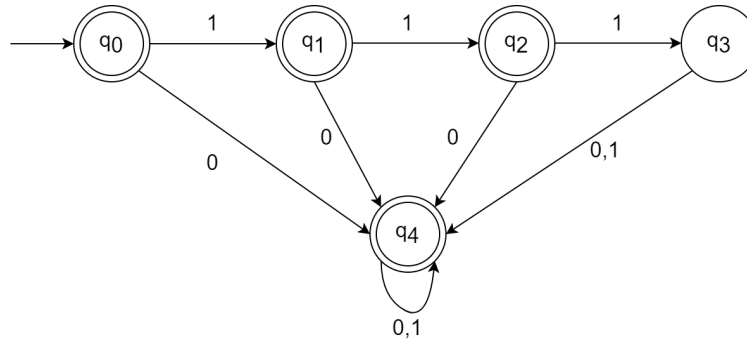


Question 1.

Given a DFA state diagram recognizing:
 $A = \{w \mid w \text{ is any binary string except } 111\}$

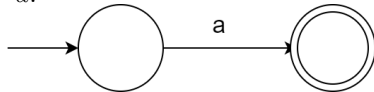


Question 2.

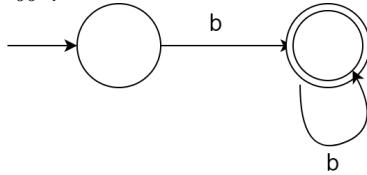
$$R = (a \cup b^+)a^+b^+ = (a \cup bb^*)aa^*bb^*$$

We can draw state diagrams for components:

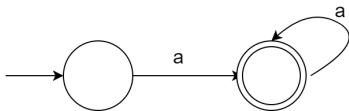
- a :



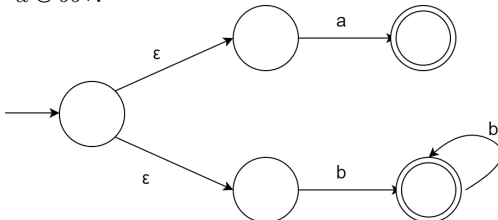
- bb^* :



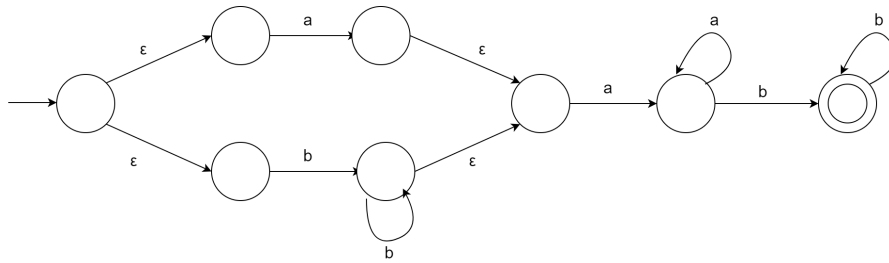
- aa^* :



- $a \cup bb^*$:



Therefore, after combination, we get the NFA for R :



Question 3.

Convert CFG to CNF:

$A \rightarrow BAB$

$B \rightarrow 00$

We do it with steps below:

1. Add start variable
 $S \rightarrow A$
 $A \rightarrow BAB$
 $B \rightarrow 00$
2. Handle rule $A \rightarrow BAB$ by add $X \rightarrow BA$
 $S \rightarrow A$
 $A \rightarrow XB$
 $X \rightarrow BA$
 $B \rightarrow 00$
3. Handle rule $B \rightarrow 00$
 $S \rightarrow A$
 $A \rightarrow XB$
 $X \rightarrow BA$
 $B \rightarrow YY$
 $Y \rightarrow 0$
4. Finally, Remove unit rule $S \rightarrow A$
 $S \rightarrow XB$
 $A \rightarrow XB$
 $X \rightarrow BA$
 $B \rightarrow YY$
 $Y \rightarrow 0$

Question 4.

$C = \{1^n 2^m 3^n 4^m \in \Sigma^* \mid n, m > 0\}$ is not context free.

We will prove it by contradiction with assumption that C is context free language with pumping length p .

Let pick $s = 1^p 2^p 3^p 4^p \in C$, $|s| = 4p > p$, following pumping lemma, we can split s in 5 part $s = uvxyz$, such that:

1. $|xy| > 0$
2. $|vxy| \leq p$. Thus, vxy can't contain more than 2 types of alphabet symbol

We consider 2 main cases:

1. vxy only consists 1 type of alphabet symbol, specifically 1s. Consider the pumping $s' = uv^i xy^i z$ with $i = 0$. Since $|xy| > 0$, s' consists k symbol 1 such that $k < p$. Then s' has the number of 1s less than number of 3s. Thus, $s' \notin C \rightarrow \text{contradiction}$. Similar for case of vxy only consists one type of 2s, 3s, 4s.
2. vxy consists 2 types of alphabet symbol, specifically 1s and 2s. Consider the pumping $s' = uv^i xy^i z$ with $i = 0$. Since $|xy| > 0$, s' has the number of 1s less than the number of 3s or the number of 2s less than the number of 4s. Thus $s' \notin C \rightarrow \text{contradiction}$. Similar for case of vxy consists (2s,3s), (3s,4s).

Therefore, C is not context free.

Question 5.

Suppose we put # at the start and \$ at the end of string on tape, the state diagram:

