Question 1

$ALL_{DFA} = \{\langle A \rangle | A \text{ is a DFA and } L(A) = \sum^*\}$.
We can show that ALL_{DFA} is decidable by design a turing machine for it:
M: on input $\langle A \rangle$, where A is a DFA

1. Design a DFA B such that $L(B) = \sum^*$
2. If $\langle A, B \rangle \in EQ_{DFA}, ACCEPT$
3. Else REJECT.

Since EQ_{DFA} is decidable, the machine M always terminates and returns the output ACCEPT/REJECT. Thus ALL_{DFA} is decidable.

Question 2

$\bar{E_{TM}}$ is recognizable since we can design a recognizer for it:
R: on input $\langle M \rangle$, where M is a Turing Machine:

1. Repeat the following for $i = 1, 2, 3, ...$
2. Run M for i steps on each input $s_1, s_2, ..., s_i$.
3. If any computation accepts, ACCEPT.

Since A_{TM} is recognizable and the set $\sum^*$ is countable, the machine R always terminates and returns ACCEPT if $\langle M \rangle \in \bar{E_{TM}}$.

Question 3

We can design a decider for it:
M: On input $\langle R \rangle$:

1. Construct $X = (0 \cup 1)^* 111 (0 \cup 1)^*$
2. Contruct $Y = R \cap X$ is also a Regular Expression
3. If $Y \in E_{RE}, REJECT$
4. Else ACCEPT.

Since E_{RE} is decidable, the machine M always terminates and returns the output ACCEPT/REJECT. Thus A is decidable.

Question 4

Prove that EQ_{CFG} is co-turing recognizable. We can design a recognizer for it.
M: on input $\langle G_1, G_2 \rangle$ is two CFGs.

1. Repeat the following for i = 1, 2, 3,...
2. with string s_i, if $\langle G_1, s_i \rangle \in A_{CFG} \neq \langle G_2, s_i \rangle \in A_{CFG}$, ACCEPT

Since A_{CFG} is decidable and the set $\sum^*$ is countable, the machine M always terminates and returns ACCEPT if $\langle G_1, G_2 \rangle \in \bar{EQ}_{CFG}$.

Question 5

$T = \{\langle M \rangle|$M is a TM that accepts w^R whenever it accepts $w\}$
Suppose that T is decidable
Firstly, we design the turing machine A such that:

$$L(A) = \begin{cases} \{01, 10\}, \text{if M accept } w \\ \{001\}, \text{otherwise} \end{cases} \quad (1)$$

A: on input string x:

1. if $x \notin \{01, 10, 001\}$ REJECT.
2. if $x \in \{01, 10\}$
   (a) if M accept w ACCEPT
   (b) else REJECT

   if $x = 001$:
   (a) if M accept w REJECT
   (b) else ACCEPT

So, we can build a decidable for A_{TM}:
R: on input $\langle M, w \rangle$:

1. Run T on $\langle A \rangle$
2. If it accept, ACCEPT
3. Else REJECT

Since T is decidable so R is decider for A_{TM}. However, A_{TM} is undecidable (Contradiction). Therefore, T is undecidable