Math 114 Exam 1
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General guidelines

- When you've solved a problem (perhaps at the back of the blue book, or on scrap paper) write your answer out neatly starting on a new page in the blue book. Make it easy for me to find each problem. Don't just circle a number. Show all units, and write complete sentences. If you've used any technology, say so.
- Remember to *read the questions carefully* before you start playing with the numbers.
- The purpose of this course is to help you learn how to use quantitative reasoning principles to solve real problems that matter to you. An exam can't test that well because you must answer the questions quickly. Here's a compromise. For homework for Thursday, rethink your answers. If you can write better ones, submit them. (Don't redo problems you got right the first time.) I will correct both the exam and the resubmissions. Getting a problem right the second time isn't worth as much as getting it right the first time, but it can make a difference in your grade. The exam is posted on the course web page at `http://www.cs.umb.edu/~eb/114/exam1/exam1.pdf` .

  Work independently. You can email me with questions, but don't consult with friends or classmates or tutors.
- The exam is "open everything" — open book, open notes, calculator. Since there are no computers in the exam room (unless you brought your own) you won't need internet access to answer the questions. If you use the internet when you redo the exam, just say so.
- Remember to show only the number of significant digits (precision) in your answer justified by the numbers you start with and the estimates you make.
- Write complete sentences. Don't use arrows and equal signs instead of the words that explain what numbers mean and what you are doing. You don't need to show me your arithmetic.

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1. (5 points)
   (a) What electronic aids will you be using? (Calculator, smartphone, computer, none?)
   (b) Read the general guidelines - particularly the ones about the form your answers should take, and the chance to improve your answers between now and Thursday. Write "I understand the instructions" as an answer for a free 5 points.
2. (20 points) Warren Buffet.

   In *The Boston Globe* on September 30, 2015 you could read that Warren Buffet is worth $62 billion, and that

   > ... if [he] gave up on aggressive investing and put his money into a simple savings account, with returns at a bare 1 percent, he'd earn more in interest each hour than the average American earns in a year.

   Use the data in this story to estimate the annual earnings of the average American. Do you think the estimate is reasonable?

3. (25 points) State's rainy day fund has dwindled over past decade,

   On October 5, 2015 *The Boston Globe* published a story with that headline that said (in part):

   > In the summer of 2007, before the massive recession began, the rainy day fund had $2.3 billion — a cushion of about 7.8 percent of total state spending, according to the Taxpayers Foundation. This summer, the rainy day balance stood at $1.1 billion — about 2.7 percent of total state spending, a fraction small enough to raise a red flag for analysts.

   (a) Calculate total state spending as of the summer of 2007 and this past summer.

   (b) What is the percentage change in total state spending between the summer of 2007 and this past summer?

   (c) Write a short argument supporting the statement "Today's rainy day fund is only about half what it was in 2007". (One good sentence will do the job.)

   (d) Write a short argument supporting the statement "Today's rainy day fund is only about a third of what it was in 2007".

   (e) Which of the previous two statements is a better description of the situation? (Tell me why you think so, don't just say it's one or the other.)

4. (25 points) Medicare fraud.

   An Associated Press story in Easton, Maryland's *The Star Democrat* on June 3, 2010 reported that

   > All told, scam artists are believed to have stolen about $47 billion from Medicare in the 2009 fiscal year, nearly triple the toll a year earlier. Medicare spokesman Peter Ashkanaz said that ... charges have been filed against 103 defendants in cases involving more than $100 million in Medicare fraud.

   (a) What percentage of the Medicare fraud has been targeted by filed charges?

   (b) What is the average claim in each fraud charge?

   (c) How many of these average size claims would need to be filed to recover the entire $47 billion?

   (d) The article suggests that the administration is vigorously pursuing Medicare fraud. Do the numbers support that suggestion?

5. (25 pointe) For-profit colleges could be banned from using taxpayer money for ads.

   In April 2012, Senators Tom Harkin and Kay Hagan introduced a Senate bill to prohibit colleges from using federal education dollars for advertising or marketing. In March 2013 they reintroduced the bill. The Senate committee on Health, Education, Labor and Pensions reported that

   - Fifteen of the largest for-profit education companies received 86 percent of their revenues from federal student aid programs — such as the G.I. Bill and Pell grants.
   - In Fiscal Year 2009, these for-profit education companies spent $3.7 billion dollars, or 23 percent of their budgets, on advertising, marketing and recruitment, which was often very aggressive and deceptive.

   (a) What were the total revenues of those fifteen for-profit colleges in fiscal year 2009?

   (b) How much did those colleges receive in federal student aid in fiscal year 2009?

   (c) How much of the money they spent on advertising, marketing and recruitment could be considered as coming from the federal government?

   (d) Could the colleges maintain the same level of advertising, marketing and recruiting expenses if they did not use any federal dollars for those purposes?

Exercise 1

Many people planned to turn in the exam questions, which suggested to me that they hadn't understood the directions. No way I could take that into account in the grading. In fact, many people didn't redo the exam at home, which surprised me.

I did take off one point (out of five) for people who didn't start question 2 on its own page (that was most of the class). **Exercise 2**

Use the data in this story to estimate the annual earnings of the average American. Do you think the estimate is reasonable?

One percent of \$62 billion is \$620 million. That's how much interest he would earn in a year. There are $24 \times 365 \approx 20 \times 400 = 8000$ hours in a year. That means Buffet would earn about

$$\frac{\$620 \text{ million}}{\text{year}} \times \frac{1 \text{ year}}{8,000 \text{ hours}} \approx \frac{\$80,000}{\text{hour}}.$$

\$80,000 sounds like more than what the average American earns in a year. It's in the right ballpark (the right number of zeroes) but I don't like it.

Doing the arithmetic more carefully the estimate comes out to \$70,729 per year. That's smaller, but I think still too high.

Looking ahead to studying averages: it just might be the mean (total earnings divided by number of earners) but saying it's for the "average American" makes it the mode or the median. The median is in fact about \$50K (lots of web sources). The mode is even less.

Exercise 3

(a) Calculate total state spending as of the summer of 2007 and this past summer.

Total state spending for the summer of 2007 was $\$2.3/0.078 = \$29.5$ billion.

Total state spending for this past summer was $\$1.1/0.027 = \$40.7$ billion.

(b) What is the percentage change in total state spending between the summer of 2007 and this past summer?

Since $40.7/29.5 = 1.38$, total state spending has increased by 38 percent.

(c) Write a short argument supporting the statement "Today's rainy day fund is only about half what it was in 2007".

Today's 1.1 billion dollar rainy day fund is less than half of the 2.7 billion dollars it was just eight years ago.

(d) Write a short argument supporting the statement "Today's rainy day fund is only about a third of what it was in 2007".

The 2.7 percent of spending in today's rainy day fund is just about one third of the 7.8 of spending it was just eight years ago.

(e) Which of the previous two statements is a better description of the situation? (Tell me why, don't just say it's one or the other.)

The rainy day fund is meant to cover unexpected expenses. It's reasonable to try to anticipate them as a fraction of spending rather than in dollar amounts. That means the size of the fund is best measured as a percentage of spending. The analysts referred to in the story think so too which is why they are concerned. Measured that way, the current fund is only a third as large as it was.

Exercise 4

(a) What percentage of the Medicare fraud has been targeted by filed charges?

$$\frac{\text{filed charges}}{\text{total fraud}} = \frac{\$100 \text{ million}}{\$47 \text{ billion}} \approx \frac{100}{50,000} = 0.002 = 0.2\%.$$

That's two tenths of one percent — \$2 out of every \$1000. Careful: it's *not* 0.002 *percent*.

(b) What is the average claim in each fraud charge?

100 million dollars total for 103 claims is about a million dollars per claim.

(c) How many of these average size claims would need to be filed to recover the entire \$47 billion?

47 billion is 47,000 million, so you'd need almost 50 thousand claims of a million dollars each to cover that amount.

(d) The article suggests that the administration is vigorously pursuing Medicare fraud. Do the numbers support that suggestion?

No. The percentage of fraud being attacked is a drop in the bucket. The effort would have to be 50 thousand times as much to get at it all.

Exercise 5

(a) What were the total revenues of those fifteen for-profit colleges in fiscal year 2009?

I know that 23% of revenue is \$3.7 billion, so

$$0.23 \times \text{ revenue} = \$3.7 \text{ billion}$$

so

$$\text{revenue} = \frac{\$3.7 \text{ billion}}{0.23} \approx \$16.2 \text{ billion}.$$

That makes sense: the answer should be about four times 3.7 since 23% is just about one fourth.

(b) How much did those colleges receive in federal student aid in fiscal year 2009?

That's easy now that I know the revenues. I just need to find 86%.

$$0.86 \times \$16.2 \text{ billion} = \$13.9 \text{ billion},$$

(c) How much of the money they spent on advertising, marketing and recruitment could be considered as coming from the federal government?

If the colleges' bookkeepers don't keep separate pockets to connect particular revenue to particular expenditure then the answer is 86% of \$3.7 billion, which is about \$3.2 billion.

(d) Could the colleges maintain the same level of advertising, marketing and recruiting expenses if they did not use any federal dollars for those purposes?

No. They spend 27% on marketing, but only 14% of their revenue isn't from the government.