Math 114 Exam 1
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General guidelines

- When you've solved a problem (perhaps at the back of the blue book, or on scrap paper) write your answer out neatly starting on a new page in the blue book. Make it easy for me to find each problem. Don't just circle a number. Show all units, and write complete sentences. If you've used any technology, say so.
- Remember to *read the questions carefully* before you start playing with the numbers.
- The purpose of this course is to help you learn how to use quantitative reasoning principles to solve real problems that matter to you. An exam can't test that well because you must answer the questions quickly. Here's a compromise. For homework for Thursday, rethink your answers. If you can write better ones, submit them. (Don't redo problems you got right the first time.) I will correct both the exam and the resubmissions. Getting a problem right the second time isn't worth as much as getting it right the first time, but it can make a difference in your grade. The exam is posted on the course web page at `http://www.cs.umb.edu/~eb/114/exam1/exam1.pdf`.

  Work independently. You can email me with questions, but don't consult with friends or classmates or tutors.
- The exam is "open everything." Google (and the internet), calculators, class notes and the text are all OK. Of course you can't use the computer to exchange email with your classmates during the exam. No text messages either, please.

  Use the web wisely, and not too much! Most of the calculations can be done with the numbers in the exam. Quick searches should find what you need from the web – don't surf around. Document your sources – "found it on google" isn't good enough.
- Remember to show only the number of significant digits (precision) in your answer justified by the numbers you start with and the estimates you make.
- Write complete sentences. Don't use arrows and equal signs instead of the words that explain what numbers mean and what you are doing. You don't need to show me your arithmetic.

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1. (5 points) Read the general guidelines - particularly the first two about the form your answers should take, and the chance to improve your answers between now and Thursday. Write "I understand the instructions" as an answer for a free 5 points.

2. (15 points) Penny Dreadful

   On page 60 of the March 31, 2008 issue of *The New Yorker* David Owen wrote

   > Breaking stride to pick up a penny, if it takes more than 6.15 seconds, pays less than the federal minimum wage.

   (a) Use the information in the quotation to figure out the minimum wage when Owen wrote his article.

   (b) Check Owen's arithmetic by comparing your answer to the actual federal minimum wage at that time. (This information is available on the web and in the text.)

3. (45 points) Taking care of your kids

   On October 4, 2014 Josh Boak of the Associated Press reported that

   > When the Great Recession struck in late 2007 and squeezed most family budgets, the top 10 percent of earners – with incomes averaging $253,146 – went in a different direction: They doubled down on their kids' futures.
   >
   > Their average education spending per child jumped 35 percent to $5,210 a year during the recession compared with the two preceding years. For the remaining 90 percent of households, such spending averaged about a flat $1,000, according to research by Emory University sociologist Sabino Kornrich. [1]

   Use the numbers in the quotation as much as you can to answer the questions that follow. When you need numbers that aren't there, estimate them, with or without the web. Be clear about any assumptions you make. If you do look things up, be sure to cite your sources.

   (a) What were the wealthiest households spending per child on education before the increase?

   (b) What percentage of their income were the the wealthiest households spending on education after the increase?

   (c) How did the percentage of their income devoted to education change when the recession started.

[1] `http://bigstory.ap.org/article/e59ee3ddaef64e2ebc75e081bf9f1d40/school-spending-affluent-widening-wealth-gap`. You won't find any more helpful information reading the article.

(d) What was the average education spending per child for all households?

(e) Estimate the 2007 total national household annual education spending on which this study is based.

(f) What does "double down" mean? Did the wealthiest households double down on education spending?

4. (35 points) Too many plastic bags.

   An article from October 3, 2007 in *The Washington Post* about how paper and plastic bags impact the environment, stated that: "Worldwide, an estimated 4 billion plastic bags end up as litter each year. Tied end to end, the bags could circle the Earth 63 times."

   (a) Are the numbers "4 billion plastic bags" and "circle the Earth 63 times" consistent?

   (b) Use an estimate of the population of the United States and some common sense to estimate how many plastic bags are used in the United States each year. Use your answer to show that the 4 billion plastic bag claim is too small by several orders of magnitude.

   (c) Confirm your U.S. estimate with a web search.

Exercise 1

Many people planned to turn in the exam questions, which suggested to me that they hadn't understood the directions. No way I could take that into account in the grading. In fact, many people didn't redo the exam at home, which surprised me.

I did take off one point (out of five) for people who didn't start question 2 on its own page (that was most of the class).

Exercise 2

(a) Use the information in the quotation to figure out the minimum wage when Owen wrote his article.

I asked Google for

0.01 dollars per 6.15 seconds in dollars per hour

and was told

(0.01 U.S. dollars) per (6.15 seconds) = 5.85365854 U.S. dollars per hour

so Owen was computing with a minimum wage of \$5.85/hour.

(b) Check Owen's arithmetic by comparing your answer to the actual federal minimum wage at that time.

The table earlier in the book confirms Owen's calculation, since the minimum wage when he wrote the article was indeed \$5.85/hour.

Exercise 3

(a) What were the wealthiest households spending per child on education before the increase?

$$\frac{\$5,210}{1.35} = \$3859.25925926 \approx \$3,860 \text{ per child.}$$

(b) What percentage of their income were the the wealthiest households spending on education after the increase?

To answer this I need to know how many children. Assuming two, on average, they were spending

$$\frac{2 \times \$5,210}{\$253146} \approx 0.0412 \approx 4.1\%.$$

(c) How did the percentage of their income devoted to education change when the recession started.

It increased by 35%.

Since $4.12/1.35 = 3.05$ it increased by about $4.12 - 3.05 = 1.07$ percentage points.

(d) What was the average education spending per child for all households?

This is a weighted average.

$$0.9 \times \$1,000 + 0.1 \times \$5,210 = \$1,421 \approx \$1,400.$$

The numbers are accidentally so simple that I could do the arithmetic in my head: $900 + 521 = 1421$. Then I rounded to \$1,400, since the \$1,000 is much less precise than the \$5,210.

(e) Estimate the 2007 total national household annual education spending on which this study is based.

My answer to the previous question tells me the average spending per child, so all I need is the number of children. If I assume 1/4 of the approximately 300 million people in the U.S. were kids the total spending was

$$\frac{\$1,400}{\text{child}} \times 75 \text{ million children} \approx \$100 \text{ billion}$$

I looked on the web to confirm some of this. The US government says about 73 million children, so my estimate was pretty good (`http://www.childstats.gov/americaschildren/tables/pop1.asp?popup=true`. These are kids aged 0-17, so not all of them needed education spending, but I won't take the time to refine my estimate further.

Searching for "education spending" led me to lots of web sites about *public* spending but only links to this research report for private spending.

(f) What does "double down" mean? Did the wealthiest households double down on education spending?

"Doubling down" means doubling a bet when gambling – usually to try to recover what you've just lost. Literally speaking, these households didn't double down on education spending.

But `http://en.wiktionary.org/wiki/double_down` gives a second meaning:

2. (idiomatic, by extension) To double or significantly increase a risk, investment, or other commitment.

They did significantly increase an investment.

Exercise 4

(a) Are the numbers "4 billion plastic bags" and "circle the Earth 63 times" consistent?

Suppose a plastic bag is about a foot long. There are about 5,000 feet in a mile, so four billion feet is 800,000 miles. I know that the distance around the earth at the equator is 25,000 miles. Then 800,000/25,000 = 800/25 = 32. So my estimate says the bags would go around the equator 32 times, not 63. If I started with the assumption that bags were two feet long I'd get the answer the article claims. But the number is in the right ballpark.

(b) Use an estimate of the population of the United States and some common sense to estimate how many plastic bags are used in the United States each year. Use your answer to show that the 4 billion plastic bag claim is too small by several orders of magnitude.

Off the top of my head: 300 million people each using 10 plastic bags a week for 50 weeks would mean 150,000 million plastic bags a year for the US. That's 150 billion, which is two orders of magnitude more than the 4 billion the article claims for the whole world. And I think 10 bags per week is a low estimate.

Since I am uncomfortable claiming the newspaper was wrong, I will check my work by reasoning backwards from the article's number. Say we use a quarter of the plastic bags in the world (even though we're just 1/20th of the population). That would be 1 billion per year for us, or just over three per year per person (since 3 * 300 million is just short of a billion). I am sure we use more than three bags per person per year!

(c) Confirm your U.S. estimate with a web search.

I googled

How Many Plastic Bags do we Use Each Year?

and found this 2008 estimate from `http://www.natural-environment.com/blog/2008/01/10/how-many-plastic-bags-do-we-use-each-year/`:

> A common estimate is that global consumption of plastic bags is over 500 billion plastic bags annually. Yes that's 500,000,000,000 plastic bags used per year. In other words, that's almost 1 million plastic bags used per minute.

That means the Washington Post estimate really was *two orders of magnitude* too small!

Lots of other web sources support this conclusion. From `http://www.salon.com/news/feature/2007/08/10/plastic_bags`:

> Every year, Americans throw away some 100 billion plastic bags after they've been used to transport a prescription home from the drugstore or a quart of milk from the grocery store. It's equivalent to dumping nearly 12 million barrels of oil.
>
> ...
>
> Only 1 percent of plastic bags are recycled worldwide – about 2 percent in the U.S.
>
> ...
>
> In the U.S., one company buys half of the used plastic bags available on the open market in the United States, using about 1.5 billion plastic bags per year. That's Trex, based in Winchester, Va., which makes composite decking out of the bags and recycled wood. It takes some 2,250 plastic bags to make a single 16-foot-long, 2-inch-by-6-inch plank.

From `http://www.copperwiki.org/index.php/Plastic_bags`:

> A million plastic bags are being consumed every minute across the globe. This amounts to 150 bags a year for every person on earth.

