

Name: \_\_\_\_\_

Math 114 Final  
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May 22, 2014

### Instructions

- Turn in your course wrapup document. (Do that now so you don't forget at the end of the exam.)
- You can find this exam at <http://www.cs.umb.edu/~eb/114/final/finals14.pdf>
- Write your answers on the exam paper whenever that's possible. Use the reverse side for more space.
- Do your Excel work in just one spreadsheet. Use separate tabs (worksheets) for separate problems.
- When you are done, email your spreadsheet to me at my gmail address

`ebolker@gmail.com`

with subject line

`Math 114 final`

Email a copy to yourself (or put your spreadsheet on your memory stick).

- Remember to show all your work, write full sentences (even paragraphs sometimes). Support your opinions with quantitative reasons.
- Round answers to the right number of significant digits.
- Use the internet only when you don't have the data you need in the problem statement.
- Please use your common sense.

1. On April 18, 2014 Leon Neyfakh wrote in *The Boston Globe* that property confiscated by the Cuban government in the 1959 revolution was

... originally valued at \$1.8 billion, which at 6 percent simple interest translates to nearly \$7 billion today. <sup>1</sup>

(a) Is the simple interest calculation in the quotation correct?

(b) What would the value be today at 6 percent compound interest?

(c) What would the value be today simply taking inflation into account?

(d) Discuss which of the three valuations makes the most sense?

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<sup>1</sup><http://www.bostonglobe.com/ideas/2014/04/18/cuba-you-owe-billion/jHAufRfQJ9Bx24TuzQyBNO/story.html>

2. Using Excel, construct a table for both a linear and an exponential model of the increase in value over time of the \$1.8 billion in property confiscated by the Cuban government in the question above.
- Set year  $t = 0$  as 1959 ( $t =$  years since 1959).
  - Linear model: the \$1.8 billion value in 1959 increases by \$108 million per year.
  - Exponential model: the \$1.8 billion value in 1959 increases by 6% per year.
- (a) Create a single properly labelled Excel chart that displays both your linear and exponential model.
- (b) Does viewing the table and chart suggest a different answer to question 1d above? If so explain why with reference to your Excel models. Put your answer here or in your spreadsheet.

3. If you follow Mass Ave from Boston to Cambridge you will cross the Charles River via the Harvard Bridge. Looking down you may notice that the bridge is marked off in *Smoots*. From one end to the other the length of the bridge is 364.4 Smoots. What is a Smoot you might ask? According to Wikipedia a Smoot is “a nonstandard unit of length created as part of an MIT fraternity prank. It is named after Oliver R. Smoot, a fraternity pledge to Lambda Chi Alpha, who in October 1958 lay on the Harvard Bridge and was used by his fraternity brothers to measure the length of the bridge.”

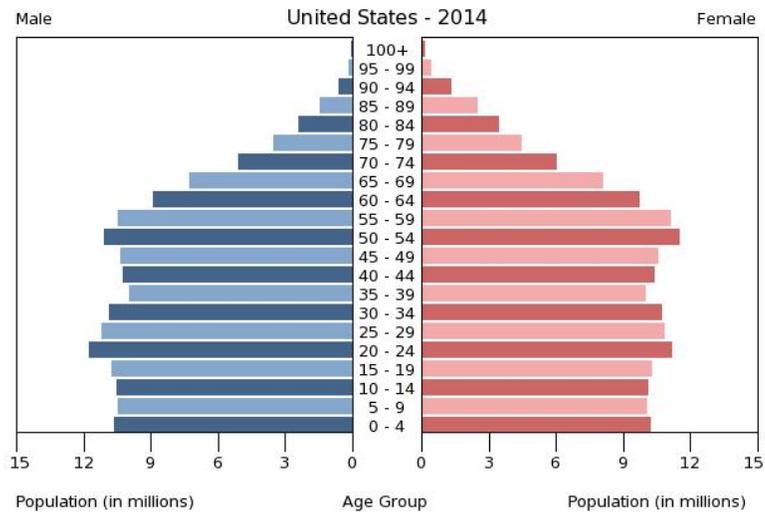
(a) The length of the Harvard Bridge in standard English measure is 2035 feet. How tall was Oliver R. Smoot, in feet and inches?

(b) How tall was Oliver R. Smoot, in meters? (Use only the right number of decimal places in your answer.)

(c) Kelly Olynyk is the 7 feet tall starting center for the Boston Celtics. How long would the Harvard Bridge be in Olynyks?

(d) What is the conversion rate between Smoots and Olynyks?

4. A Viagra ad on TV stated that more than 20 million men already use Viagra. Use the US population pyramid below to argue whether or not this claim seems reasonable. Be explicit about any assumptions you have made about the age groups of men who might typically use this drug.



5. The Math SAT scores for last years entering freshman class of 1412 students is given in the table below:

MSAT Score	Percent
450-499	32%
500-549	16%
550-599	12%
600-649	24%
650-699	8%
700-749	8%

- (a) Construct a properly labeled histogram in Excel using this data.
- (b) Estimate the mode, median and mean MSAT scores for these students. Show your calculations, either on this paper or as formulas in your spreadsheet.
- (c) How many students had an MSAT score higher than the median?

6. According to Vox.com, “the top 25 hedge fund managers earned a collective \$21.1 billion this year.” Vox.com put this figure into context, saying: “it’s about 0.13 percent of total national income for 2013 being earned by something like 0.00000008 percent of the American population.”

(a) Use the data from Vox.com to estimate the total national income in 2013.

(b) The same Vox.com article stated that the earnings of the 25 hedge fund managers was “about 2.5 times the income of every kindergarten teacher in the country combined.” Use this information to estimate how many kindergarten teachers there were in 2013. To help you answer this question, use the chart below (from yet another Vox.com article that compared teachers’ and lawyers’ pay). Show all your calculations.

### Teachers get appreciated, bankers & lawyers get paid



Source: BLS



(c) Can you verify Vox.com’s estimated income comparison of hedge fund managers and kindergarten teachers by using the Internet? Show your calculations and fully explain your answer.

7. Here's a made up story. The dean at a fancy private high school is very worried. She suspects that about 20% of the 1000 students on campus are using drugs. She has asked all the parents to administer a home drug test to their kids (since it's a private school she can actually require them to do it). She has read on the web that

With home drug testing methods believed to produce reliable and accurate results, many of us overlook the cases of false positives and draw conclusions on the suspect before reconfirming the result. But, researchers from the Boston University have found out that drug tests may produce false positives in 5-10% of cases and false negatives in 10-15% of cases. <sup>2</sup>

(I found several blogs that seem to report on this same study. None gives a link or a precise reference. I haven't been able to locate the original.)

Answer the following questions, assuming the worst cases (10% false positive rate, 15% false negative rate).

- (a) Build the contingency table for this drug screening scenario. To do that you will have to figure out
- How many students are drug users.
  - How many of the drug users test positive. How many test negative.
  - How many students are drug free.
  - How many of the drug free students test positive. How many test negative.

You may do the arithmetic with your calculator, or use the spreadsheet at <http://www.cs.umb.edu/~eb/114/final/ContingencyTable.xlsx>. If you use the spreadsheet, explain how the formula in cell C10 does what it is supposed to.

Use the other side of the paper for the rest of the questions:

- (b) What is the true positive rate?
- (c) Student John Smith tested positive. What is the probability that he is really on drugs?
- (d) Student Jane Doe tested negative. What is the probability that she is really drug free?
- (e) Answer the previous two questions if you assume the best cases for reported false values in the Boston University study.

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<sup>2</sup><http://lapoliticaesotracoa.blogspot.com/2012/05/how-to-avoid-false-positives-while.html>

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### Exercise 1

- (a) Is the simple interest calculation in the quotation correct?

At 6 percent simple interest the value grows linearly at a rate (slope) of  $0.06 \times 1.8 = 0.108$  billion dollars per year. In the 55 years from 1959 to 2014 it would grow to

$$\$1.8\text{b} + 0.108 \frac{\text{\$b}}{\text{year}} \times 55 \text{ years} = \$7.74\text{b}$$

so the quotation seems to underestimate the 2014 value. It's almost 8 billion dollars.

- (b) What would the value be today at 6 percent compound interest?

Compounding at a rate of 6 percent per year the value after 55 years would be

$$\$1.8\text{b} \times 1.06^{55} = \$44.4\text{b}.$$

- (c) What would the value be today simply taking inflation into account?

The inflation calculator tells me that \$1.8 billion in 1959 would be worth \$14.2 billion in 2014.

- (d) Discuss which of the three valuations makes the most sense?

Simple interest isn't a realistic way to think about how the value of an investment changes over time. The "six percent" in the compound interest calculation is a made up growth factor. I think the most sensible way to think about what that 1.8 billion dollars would be worth in 2014 is simply to adjust it for inflation.

I could make a case for something larger than the inflation adjusted figure.

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### Exercise 2

- (a) Create a single properly labelled Excel chart that displays both your linear and exponential model.

I did that by adding some rows to the data in the linear vs exponential spreadsheet that comes with the text.

- (b) Does viewing the table and chart suggest a different answer to questions 1d above? If so explain why with reference to your Excel models.

I didn't learn anything new from Excel. The answers matched mine in the last question.

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### Exercise 3

- (a) The length of the Harvard Bridge in standard English measure is 2035 feet. How tall was Oliver R. Smoot, in feet and inches?

$$\frac{2035 \text{ feet}}{364.4 \text{ Smoots}} = 5.58452250274 \frac{\text{feet}}{\text{Smoot}}.$$

That decimal fraction is a pretty ugly number of feet – and 5.6 feet is *not* 5 feet 6 inches.

Multiplying by 12 gives me almost exactly 67 inches per Smoot. So Smoot was 5 feet 7 inches tall.

Wikipedia provides that answer too – but the question called for your own calculation.

The Google calculator knows about Smoots. Asking for “one smoot in inches” tells me

$$\boxed{1 \text{ smoot} = 67 \text{ inches}}$$

- (b) How tall was Oliver R. Smoot, in meters? (Use only the right number of decimal places in your answer.)

According to google, one smoot is 1.7018 meters. That’s best reported as 1.7 meters.

- (c) Kelly Olynyks is the 7 feet tall starting center for the Boston Celtics. How long would the Harvard Bridge be in Olynyks?

$$2035 \text{ feet} \times \frac{1 \text{ Olynyk}}{7 \text{ feet}} = 290.7 \text{ Olynyks}.$$

- (d) What is the conversion rate between Smoots and Olynyks?

$$\frac{1 \text{ Olynyk}}{1 \text{ Smoot}} = \frac{84 \text{ inches}}{67 \text{ inches}} = 1.25$$

so there are 1.25 Smoots per Olynyk, or, if you prefer,  $1/1.25 = 0.80$  Olynyks per Smoot.

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#### Exercise 4

The population pyramid tells me there are about  $11 + 10 + 8 + 4 + 3 + 2 = 38$  million men between the ages of 60 and 95. If I assume that covers pretty much all the potential Viagra customers the ad claims that half of them actually use the drug. That sounds like an overestimate to me, but it’s probably in the right ballpark.

I think that’s a reasonable answer to the question using just the data in the question. Many students went to the web for more information, and found it. Many sites suggest that lots of younger men use Viagra too, at least occasionally. If I think of the potential user population as half the 160 million men in the country then the ad claims that  $1/4$  of them are actual users. That might well be true.

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#### Exercise 5

(a) Construct a properly labeled histogram in Excel using this data.

Done.

(b) Estimate the mode, median and mean MSAT scores for these students. Show your calculations, either on this paper or as formulas in your spreadsheet.

The mode is the highest bar corresponding to a score of 450-499.

The first two categories account for  $32 + 16 = 48$  percent of the students. That's very nearly half, so about half the students have a score of 549 or less. The median score is about 550.

I did the mean the way the book does, with calculations in Excel. The answer is a score of 567.

(c) How many students had an MSAT score higher than the median?

By definition, that's half the students, or 706.

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## Exercise 6

(a) Use the data from Vox.com to estimate the total national income in 2013.

Vox says total income is  $(\$21.1 \text{ billion})/0.0013 \approx \$16 \text{ trillion}$ .

(b) The same Vox.com article stated that the earnings of the 25 hedge fund managers was "about 2.5 times the income of every kindergarten teacher in the country combined." Use this information to estimate how many kindergarten teachers there were in 2013, To help you answer this question, use the chart below (from yet another Vox.com article that compared teachers and lawyers pay). Show all your calculations.

Kindergarten teachers make about \$50 thousand per year. Vox.com claims

$$2.5 \times \$50,000 \times (\text{number of kindergarten teachers}) \approx \$21.1 \text{ billion},$$

which says there are about 170,000 kindergarten teachers.

(c) Can you verify Vox.com's estimated income comparison of hedge fund managers and kindergarten teachers by using the Internet? Show your calculations and fully explain your answer.

The population pyramid on this exam says there are about 20 million 5-9 year olds in the US. That means about 4 million 5 year olds. If they all were in kindergarten classes each with 20 kids that would call for  $(4 \text{ million})/20 = 200,000$  teachers. The Vox.com estimate is clearly in the right ballpark.

Looking for "number of kindergarten teachers" on the web led me to the Bureau of Labor Statistics site <http://www.bls.gov/ooh/education-training-and-library/child-care-and-elementary-school-teachers.htm> where I found that in 2012 there were 1,519,700 kindergarten and elementary school teachers. I'd have to look harder to sort them by grade, but I'm comfortable with the 170 to 200 thousand kindergarten teachers I found two ways above.

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## Exercise 7

(a) Build the contingency table for this drug screening scenario. To do that you will have to figure out

- How many students are drug users: 20% of 1000, so 200.
- How many of the drug users test positive. How many test negative.  
170 of the 200 users test positive. The other 30 test negative (these are the false negatives).
- How many students are drug free. The other 800.
- How many of the drug free students test positive. How many test negative.  
720 of the 800 clean students test negative. 80 are false positives.

You may do the arithmetic with your calculator, or use the spreadsheet at <http://www.cs.umb.edu/~eb/114/final/ContingencyTable.xlsx>. If you use the spreadsheet, explain how the formula in cell C10 does what it is supposed to.

Cell C10 is for the true positives. The formula there is  $=C4*C5*(1-C6)$ . The first two factors multiplied together give the total number of drug users (population  $\times$  incidence rate). The true positive probability is  $(1 - \text{false positive probability})$  – the last factor.

(b) What is the true positive rate?  $100\% - 10\% = 90\%$ .

(c) Student John Smith tested positive. What is the probability that he is really on drugs?  
That's  $170/250 = 0.68$ . In Excel it's  $=C10/E10$ .

(d) Student Jane Doe tested negative. What is the probability that she is really drug free?  
 $760/780 = 96\%$ .

(e) Answer the previous two questions if you assume the best cases for reported false values in the Boston University study.

82% and 97% – Excel did the work for me.