# History of Mathematics
# Homework 1 Solution

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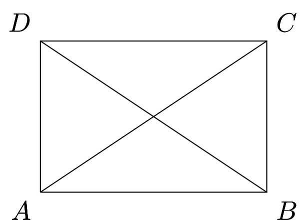
February 7, 2014

1. Sacchieri's quadrilateral (Dori Gilbert's solution) [1]

   **Proposition.** *Let $ABCD$ be a quadrilateral figure having right-angles at $A$ and $B$ with equal sides $AD$ and $BC$. I say that the angles at $D$ and $C$ are equal.*

   Let the straight line lines $AC$ be joined from the point $A$ to the point $C$ and the straight line $BD$ from the point $B$ to the point $D$ [post. 1].

   Then since the straight line $AD$ equals the straight line $CB$ and the angle $DAB$ equals the angle $CBA$ [post. 4], and certainly the straight line $AB$ equals $BA$. Then the triangles $DAB$ and $CBA$ are equal and the straight line $BD$ equals $AC$ [prop 1.4]

   Now angle $DAB$ minus angle $CAB$ equals angle $DAC$ and angle $CBA$ minus angle $DBA$ equals angle $CBD$, so angles $DAC$ and $CBD$ are equal [C.N. 3]

   Since straight line $DA$ equals straight line $CB$ and straight line $BD$ equals straight line $AC$ and angle $DAC$ equals angle $CBD$, triangle $DCB$ equals triangle $ADC$ [prop 1.4]. Thus the angles at $D$ and $C$ are equal, which is the very thing it was required to do.

   *Note:* Several of you attempted proofs that used parallelism in some way. I did not read them. They miss the whole point of the exercise, which is

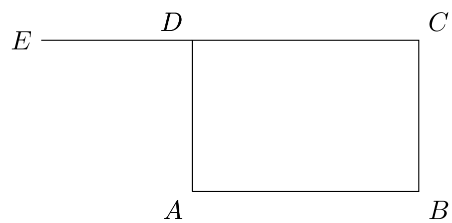
[1] I will find a good way for the class to submit TEX source online, so I don't have to retype solutions when I want to share them.

to show that those angles are equal *whether or not* the fifth postulate is true. Dori's proof uses *only* Proposition 1.4, which does not depend on Postulate 5.

2. Prove that with the parallel postulate, $C$ and $D$ in Sacchieri's quadrilateral are right angles. This is Exercise 1.4 in the text. The hint there suggests using Proposition 27.

   Here are two proof, in (relatively) modern style.

   - Extend line $CD$ to $E$.

     Now Proposition 1.27 says $AD$ and $BC$ are parallel. Then Proposition 29 (which uses the parallel postulate) says angles $EDA$ and $ECB$ are equal. But we've just proved $ECB$ equals $CDA$, so the two angles at $D$ are equal. Definition 10 says they are both right angles.
   - I know that the parallel postulate implies that the angles of any triangle sum to two right angles. The line $AC$ splits the Sacchieri quadrilateral into two triangles, so the angle sum for the quadrilateral is four right angles. Two of those are at the bottom. Since I've proved that the two at the top are equal, each must be a right angle.

LaTeX source, so you can see how it's done.

```
% Math 370 hw1
%
\documentclass{article}
\pagestyle{empty}

\usepackage{amsmath}
\usepackage{amsthm}
\usepackage{hyperref}
\usepackage{graphicx}
\usepackage{verbatim}
\usepackage{tikz} % for figures

%% create an environment for propositions
\newtheorem*{prop}{Proposition}

\newcommand{\coursehome}
{http://www.cs.umb.edu/~eb/370}

\title{History of Mathematics \\
Homework 1 Solution
}
\author{Ethan Bolker}

\begin{document}

\maketitle

\begin{enumerate}

\item  Sacchieri's quadrilateral (Dori Gilbert's solution)
\footnote{I will find a good way for the class to submit \TeX{} source
  online, so I don't have to retype solutions when I want to share them.}

\begin{prop}
Let $ABCD$ be a quadrilateral figure having right-angles at $A$ and
$B$ with equal sides $AD$ and $BC$. I say that the angles at $D$ and
$C$ are equal.
\end{prop}

%\begin{center}
%\includegraphics[width=0.6\textwidth]{sacchieri} % TeX will find the png
%\end{center}

Let the straight line lines $AC$ be joined from the point $A$ to the
```

point $C$ and the straight line $BD$ from the point $B$ to the point
$D$ [post. 1].

```
\newcommand{\ptA}{(0,0) node [below left] {$A$}}
\newcommand{\ptB}{(3,0) node [below right] {$B$}}
\newcommand{\ptC}{(3,2) node [above right] {$C$}}
\newcommand{\ptD}{(0,2) node [above left] {$D$}}

\begin{center}
\begin{tikzpicture}
\draw \ptA -- \ptB -- \ptC -- \ptD -- \ptA;
\draw \ptA -- \ptC;
\draw \ptB -- \ptD;
\end{tikzpicture}
\end{center}
```

Then since the straight line $AD$ equals the straight
line $CB$ and the angle $DAB$ equals the angle $CBA$ [post. 4], and
certainly the straight line $AB$ equals $BA$. Then the triangles $DAB$
and $CBA$ are equal and the straight line $BD$ equals $AC$ [prop 1.4]

Now angle $DAB$ minus angle $CAB$ equals angle $DAC$ and angle $CBA$
minus angle $DBA$ equals angle $CBD$, so angles $DAC$ and $CBD$ are
equal [C.N. 3]

Since straight line $DA$ equals straight line $CB$ and straight line
$BD$ equals straight line $AC$ and angle $DAC$ equals angle $CBD$,
triangle $DCB$ equals triangle $ADC$ [prop 1.4]. Thus the angles at
$D$ and $C$ are equal, which is the very thing it was required to do.

\emph{Note:} Several of you attempted proofs that used parallelism in
some way. I did not read them. They miss the whole point of the
exercise, which is to show that those angles are equal \emph{whether
or not} the fifth postulate is true. Dori's proof uses \emph{only}
Proposition 1.4,
which does not depend on Postulate 5.

\item Prove that with the parallel postulate, $C$ and $D$ in
Sacchieri's quadrilateral are right angles. This is Exercise 1.4 in
the text. The hint there suggests using Proposition 27.

Here are two proof, in (relatively) modern style.

```
\begin{itemize}

\item Extend line $CD$ to $E$.

\newcommand{\ptE} {(-2,2) node [left] {$E$}}
\begin{center}
\begin{tikzpicture}
\draw \ptA -- \ptB -- \ptC -- \ptD -- \ptA;
\draw \ptE -- \ptD;
\end{tikzpicture}
\end{center}

Now Proposition 1.27 says $AD$ and $BC$ are parallel. Then
Proposition 29 (which uses the parallel postulate) says angles $EDA$
and $ECB$ are equal. But we’ve just proved $ECB$ equals $CDA$, so the
two angles at $D$ are equal. Definition 10 says they are both right
angles.

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  any triangle sum to two right angles. The line $AC$ splits the
  Sacchieri quadrilateral into two triangles, so the angle sum for the
  quadrilateral is four right angles. Two of those are at the
  bottom. Since I’ve proved that the two at the top are equal, each
  must be a right angle.


\end{itemize}


\end{enumerate}

\newpage

\LaTeX{} source, so you can see how it’s done.

\verbatiminput{hw1Solution}

\end{document}
```