History of Mathematics
Homework 3

Ethan Bolker

February 15, 2014

This homework is due Thursday, February 20.

1. Let P and Q be two points in the unit disk. Find a Euclidean construction for the (hyperbolic/pseudo) line joining them. That is, find a straightedge-and-compass construction for the circle through those points that meets the unit circle at a right angle. (This is not easy. After trying it yourself (or with a classmate) feel free to try to find a solution on the web. If you do, you should write it out in your own words (and pictures) and tell me where and how you found it.)

2. Calculate the (hyperbolic/pseudo) distance from the center of the unit disk to a point at (Euclidean) distance $r < 1$ from the center.

   Recall that distances at a point P at distance $r < 1$ from the center of the unit disk are stretched by a factor $1/(1 - r^2)$.

   Hint. Your argument should lead to an integral that you can evaluate using what you learned in Calculus. If you don't remember what you need you should be able to look it up.

3. Use your answer to the previous problem to show that lines through the center are infinitely long - in other words, that the distance from the center approaches ∞ as $r \to 1$.

4. At `http://math.stackexchange.com/questions/675522/whats-the-intuition-behind-pythagoras` you will find a question and several answers about the Pythagorean Theorem. I read through them rapidly, but found no mention of the fact that the theorem depends on the parallel postulate.

   I think we should provide another answer to point that out. (I have enough reputation to do that.) For this homework, write something that I might post.

   Here's a hint. What's a square in spherical or hyperbolic geometry? Can you compute the area of a square by squaring the length of its side?

5. Build a paper model of the hyperbolic plane. There are instructions at `http://euler.slu.edu/escher/index.php/Hyperbolic_Paper_Exploration`.

- Answer the two questions there.
- Let ABC be one of the equilateral triangles you taped together to make your hyperbolic plane. Let D be the foot of the perpendicular from C to AB. Try to compute the angle of parallelism $\Pi(CD)$. (I do not know the answer. I don't even know whether the question makes sense. You should be able to find some upper and lower bounds for that angle by experiment.)

Bring your model to class if you can. That might be hard to do, since you can't flatten it out!

Here is the LaTeX source for this document. You can cut it from the pdf and use it to start your answers. I used the `\jobname` macro for the source file name, so you can call your file by any name you like.

```
% Math 370 hw3 Spring 2014
%
\documentclass{article}
\pagestyle{empty}

\usepackage{amsmath}
\usepackage{amsthm}
\usepackage{hyperref}
\usepackage{graphicx}
\usepackage{verbatim}

%% create an environment for theorems
\newtheorem*{thm}{Theorem}

\newcommand{\coursehome}
{http://www.cs.umb.edu/~eb/370}

\title{History of Mathematics \\
Homework 3
}
\author{Ethan Bolker}

\begin{document}

\maketitle

This homework is due Thursday, February 20.

\begin{enumerate}

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  construction for the (hyperbolic/pseudo) line joining them. That is,
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  those points that meets the unit circle at a right angle. (This is
  not easy. After trying it yourself (or with a classmate) feel free
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center.

Recall that distances at a point $P$ at distance $r < 1$ from the
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\emph{Hint.} Your argument should lead to an integral that you can
evaluate using what you learned in Calculus. If you don’t remember
what you need you should be able to look it up.

\item
Use your answer to the previous problem to show that lines through the
center are infinitely long - in other words, that the distance from
the center approaches $\infty$ as $r \rightarrow 1$.

\item At
  \url{http://math.stackexchange.com/questions/675522/whats-the-intuition-behind-pythagoras-
  you will find a question and several answers about
the Pythagorean Theorem. I read through them rapidly, but found no
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Here’s a hint. What’s a square in spherical or hyperbolic geometry?
Can you compute the area of a square by squaring the length of its
side?

\item Build a paper model of the hyperbolic plane. There are
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\begin{itemize}
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\item Let $ABC$ be one of the equilateral triangles you taped together
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\newpage

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\verbatiminput{\jobname}

\end{document}
```