History of Mathematics
Homework 4

Ethan Bolker

March 1, 2014

This homework is due on Thursday, March 6 .

Note: I've used some new (to you) LaTeX idioms in this document. Learn them. Make sure to submit LaTeX source with your assignment – preferably as part of the pdf, as here.

1. Write a short essay synthesizing what you learned about mathematics and the history of mathematics from our work on the parallel postulate. Please make this interesting for me to read. Don't just write a summary with names, dates and theorems. Consider telling me what was hard, what was easy, what was fun, what you would have liked more or less of, how the material connects to what you knew and to what you hope to know.

2. Positional notation

   In class this week we stumbled into a discussion of positional notation in our exploration of the history of the search for big primes.

   (a) The invention of positional notation is an important milestone in the history of mathematics.[1] Find out a little bit about it, and write a paragraph or two about something you find particularly interesting. You can start at the wikipedia page (`http://en.wikipedia.org/wiki/Positional_notation` but then dig deeper about some part that intrigues you.

   (b) Read the short essay at `http://www.cs.umb.edu/~eb/370/hw4/ssegm.pdf` (TeX source at `http://www.cs.umb.edu/~eb/370/hw4/ssegm.tex`). If you're so inclined, do the homework there.

   (c) Learn about the "Russian peasant" multiplication algorithm. Understand how its proof depends on the binary expansion of the factors. Work a pretty example.

3. Write a little bit about the history of logarithms. Were they *invented* or *discovered*? Write down some arguments for *both sides* of that question.

[1] Should that sentence read "The inventions . . . are important milestones . . . "?

4. The algebraic identity

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \cdots + ab^{n-2} + b^{n-1}) \qquad (1)$$
$$= \sum_{k=0}^{n-1} a^{n-1-k}b^k$$

is one of the most important in mathematics. When $n = 2$ it's the well known identity for the difference of two squares. We'll see soon how Euclid proved that case – with geometry, of course, not algebra.

   (a) Prove identity 1.

   (b) Use identity 1 to prove that $2^N - 1$ can be prime (hence a Mersenne prime) only if N is prime. (This is Exercise 4.3 in the text.)

   You *should* be able to figure out a proof. If you can't, consider this hint: How might you factor $a^{xy} - 1$? If you must, you can look up a proof and reproduce it in your own words.

   (c) Use identity 1 to prove that $2^N + 1$ can be prime (hence a Fermat prime) only if N is a power of 2.

   Hint: A number is a power of 2 if and only if it has no odd factors.

5. On page 172 our text mentions the Pythagorean triple $(12709, 13500, 18541)$ known to the Babylonians. Show how to construct it using the characterization of primitive Pythagorean triples as $(m^2 - n^2, 2mn, m^2 + n^2)$.

6. Modify Euler's argument from our text to prove that there are no non-trivial integral solutions to the equation

$$x^4 - y^4 = z^4.$$

   This is an exercise in the book. You can use modern notation, or mimic Euler's style.

Here is the LaTeX source for this document. You can cut it from the pdf and use it to start your answers. I used the `\jobname` macro for the source file name, so you can call your file by any name you like.

```
% Math 370 hw4 Spring 2014
%
\documentclass{article}
\pagestyle{empty}

\usepackage{amsmath}
\usepackage{amsthm}
\usepackage{hyperref}
\usepackage{graphicx}
\usepackage{verbatim}

%% create an environment for theorems
\newtheorem*{thm}{Theorem}

\newcommand{\coursehome}
{http://www.cs.umb.edu/~eb/370}

\title{History of Mathematics \\
Homework 4
}
\author{Ethan Bolker}

\begin{document}

\maketitle

This homework is due on Thursday, March 6 .

Note: I've used some new (to you) \LaTeX{} idioms in this
document. Learn them. Make sure to submit \LaTeX{} source with your
assignment -- preferably as part of the pdf, as here.

\begin{enumerate}

\item
Write a short essay synthesizing what you learned about
mathematics and the history of mathematics from our work on the
parallel postulate. Please make this interesting for me to read. Don't
just write a summary with names, dates and theorems. Consider telling
me what was hard, what was easy, what was fun, what you would have
liked more or less of, how the material connects to what you knew and
to what you hope to know.
```

```
\item Positional notation

In class this week we stumbled into a discussion of positional
notation in our exploration of the history of the search for big
primes.

\begin{enumerate}

  \item
The invention of positional notation is an important milestone in the
history of mathematics.%
\footnote{Should that sentence read ‘‘The inventions \ldots are
  important milestones \ldots’’?
}
Find out a little bit about it, and write a
paragraph or two about something you find particularly
interesting. You can start at the wikipedia page
(\url{http://en.wikipedia.org/wiki/Positional_notation} but then dig
deeper about some part that intrigues you.

\item Read the short essay at \url{\coursehome/hw4/ssegm.pdf} (\TeX{}
  source at \url{\coursehome/hw4/ssegm.tex}). If you’re so inclined,
  do the homework there.

\item Learn about the ‘‘Russian peasant’’ multiplication
  algorithm. Understand how its proof depends on the binary expansion
  of the factors. Work a pretty example.

\end{enumerate}

\item Write a little bit about the history of logarithms. Were
 they \emph{invented} or \emph{discovered}? Write down some arguments
  for \emph{both sides} of that question.

\item The algebraic identity
% Leaving an empty line here makes the TeX easier to read. Making it
% a comment tells TeX not to start a new paragraph.
%
\begin{align}\label{eq:anbn}
  a^n - b^n & = (a-b)(a^{n-1} + a^{n-2}b + \cdots + ab^{n-2} + b^{n-1}) \\
         & = \sum_{k=0}^{n-1} a^{n-1-k} b^k \notag
\end{align}
%
is one of the most important in mathematics. When $n=2$ it’s the well
known identity for the difference of two squares. We’ll see soon how
```

```
Euclid proved that case -- with geometry, of course, not algebra.

\begin{enumerate}
\item Prove identity~\ref{eq:anbn}.

\item Use identity~\ref{eq:anbn} to prove that $2^N -1$ can be prime (hence a
  Mersenne prime) only if $N$ is prime. (This is Exercise 4.3 in the text.)

  You \emph{should} be able to figure out a proof. If you can't,
  consider this hint: How might you factor $a^{xy} - 1$? If you must,
  you can look up a proof and reproduce it in your own words.

\item Use identity~\ref{eq:anbn} to prove that $2^N +1$ can be prime (hence a
  Fermat prime) only if $N$ is a power of 2.

  Hint: A number is a power of 2 if and only if it has no odd factors.

\end{enumerate}

\item On page 172 our text mentions the Pythagorean triple
  $(12709,13500,18541)$ known to the Babylonians. Show how to construct
  it using the characterization of primitive Pythagorean triples as
  $(m^2 - n^2,2mn,m^2+n^2)$.

\item Modify Euler's argument from our text to prove that there are no
  nontrivial integral solutions to the
  equation
%
\begin{equation*}
x^4 - y^4 = z^4 .
\end{equation*}

This is an exercise in the book. You can use modern notation, or mimic
Euler's style.
\end{enumerate}


\newpage

Here is the \LaTeX{} source for this document. You can cut it from the
pdf and use it to start your answers. I used the \verb!\jobname! macro
for the source file name, so you can call your file by any name you like.

\verbatiminput{\jobname}

\end{document}
```