SSEGM

Ethan D. Bolker
Carrie Hollingsworth
Grace Biondi
Solomon Bixby
Sam Feuer
Max Schleifer
Eva Steinitz

several years ago

May 21, 2010 – naming numbers[1]

"For a change, let's all work together, and, for a change, try to pay attention from time to time rather than rushing excitedly off to try your own ideas as soon as you get them."

All (reluctantly): "OK."

"We'll start by naming the numbers, and choosing a symbol to write for each one. For the first few we have the usual names and symbols:

0 1 2 3 4 5 6 7 8 9

We don't want to write '10' next, since that uses two of the symbols over again."

Sam: "How about B?"

[1] I wanted to introduce duodecimal arithmetic. When I've tried this before with other groups of kids (math club at the Manning, Sam once) I discovered that one of the major problems was distinguishing between numbers and their names. Since I'd had occasion earlier with this group to see how excited they were by choosing names for mathematical entities, I started with that.

"Good idea.[2] Then we could continue this way

0 1 2 3 4 5 6 7 8 9 B C D E . . .

But that's boring. Let's make up symbols instead."

Eva: "How about a circle with a dot in it for after 9? We can call it 'cirdot'?"

I write $\odot$. Eva salutes Sir Dot. I write "sirdot" with an 's' rather than a 'c'.

Grace: "'Pie' next." She sketches a complex picture of a tall apple pie.

"I think that will take too long to draw each time we need it. Symbols should be easy to write. How about a piece of pie instead?" I draw Δ.

Grace: "Let's call it 'apple', not 'pie'."

Max: "We'll have 'sirpent' next." He draws $\pentagon$.

Sam (picking up on "serpent"): "Then we can have a snake. It can be just **S**."

Next we got a stick figure, "sirbob," and a light bulb, "sirbulb."

We could clearly have gone on happily punning new knight number names. But I wanted to move on to mathematics, so I posed some arithmetic questions to which they rapidly provided the answers:

$$3 + 5 = 8$$

$$6 + 5 = \Delta$$

$$8 + 5 = \mathbf{S}$$

$$\odot + 2 = \pentagon$$

Then I asked for

$$\odot + \pentagon$$

and it dawned on everyone (particularly Eva) that we would need more names – in fact, names forever, one for each number.

I asked how we managed everyday arithmetic with just ten symbols.

"After 9 we write 10."

"Why do you think we start naming the numbers with two digits after we reach 9?"

[2] I wondered why he started with 'B' rather than 'A', and was tempted to revise his suggestion so that we could get to standard hexadecimal later on, but decided just to let it run.

"Because we have just 10 fingers."

"So if we had 6 fingers on each hand we'd have invented two more single digit symbols before we moved to two digit numbers. Perhaps

$$0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ \odot\ \Delta$$

Then how would you write the next number?"

All agree on a '1' followed by a '0'.

"And what would you call it?"

Sam blurts out "ten" and I admonish him: "No ordinary names for the numbers beyond 9, or we'll soon be very confused!"

Grace: "We can call it 'table'."[3]

Then we know how to keep counting: "table 1, table 2 and so on. What's after "table 9"?

All: "table sirdot, table apple, two table." I write down

$$19,\ 1\odot,\ 1\Delta,\ 20.$$

Saying the numbers before writing them guaranteed that they'd say "two table" and not "twenty" for 20.[4]

They decide they want to name the whole number system as well as naming the numbers themselves. Someone suggests "SEGM" for "Sam, Eva, Grace, Max" and all agree.

The addition table

It's time to be systematic and write out SEGM's addition table. They each start with an empty table of the right size in their math notebooks and begin to fill it in.

+	0	1	2	3	4	5	6	7	8	9	$\odot$	Δ
0	0	1	2	3	4	5	6	7	8	9	$\odot$	Δ
1	1	2	3	4	5	6	7	8	9	$\odot$	Δ	10
2	2	3	4	5	6	7	8	9	$\odot$	Δ	10	11
...												

[3] I thought of suggesting "douze", reminiscent of "dozen", but decided not to interrupt whatever they chose to invent.

[4] I did quietly point out the likely etymology of the English "twenty" as "two ten" but no one was paying attention, which was just as well.

They know that the columns will be the same as the rows. They quickly see the constant diagonals that slope down to the left. Then they busily fill in the numbers without further thought. I suggested we move on to something more interesting.[5]

The multiplication table

This begins innocently enough. The first two rows are easy. They have fun with the third row when they see the pattern. Most realize quickly that counting by twos rather than multiplying is the faster way to get the next entry.

$\times$	0	1	2	3	4	5	6	7	8	9	$\odot$	Δ
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	$\odot$	Δ
2	0	2	4	6	8	$\odot$	10	12	14	16	18	$1\odot$
...												

Max says "sirdot is an even number!" I applaud. [6]

The rows for 3 and 4 are harder, but still show patterns:

$\times$	0	1	2	3	4	5	6	7	8	9	$\odot$	Δ
...												
3	0	3	6	9	10	13	16	19	20	23	26	29
4	0	4	8	10	14	18	20	24	28	30	34	38
...												

[5] I think they would still have to pause for thought to compute $\odot + \odot$, but decide not to spend more time on the full table now.

[6] Actually, he said apple was an even number, and at the time I didn't see his error. But the applause is warranted despite the error. What was important was his realization that Δ and $\odot$ were just numbers with the usual properties of numbers, independent of what they were called. He clearly understood "even" as a property divorced from the last digit in decimal representation.

They find the row for 5 weird and difficult. No pattern at all. Eva said, with surprise and pleasure, counting on her fingers (just ten of them) “This is hard!” But the row for 6 is a reward.

$\times$	0	1	2	3	4	5	6	7	8	9	$\odot$	Δ
...												
5	0	5	$\odot$	13	18	21	26	2Δ	34	39	42	47
6	0	6	10	16	20	26	30	36	40	46	50	56
...												

Looking over Eva’s shoulder, I show her that counting by 6’s in SEGM is just like counting by 5’s in ordinary English.

I ask them to finish the multiplication table for homework.[7]

Here it is with one more row:

$\times$	0	1	2	3	4	5	6	7	8	9	$\odot$	Δ
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	$\odot$	Δ
2	0	2	4	6	8	$\odot$	10	12	14	16	18	1$\odot$
3	0	3	6	9	10	13	16	19	20	23	26	29
4	0	4	8	10	14	18	20	24	28	30	34	38
5	0	5	$\odot$	13	18	21	26	2Δ	34	39	42	47
6	0	6	10	16	20	26	30	36	40	46	50	56
7	0	7	12	19	24	2Δ	36	41	48	53	5$\odot$	65
8	0	8	14	20	28	34	40	48	54	60	68	74
9	0	9	16	23	30	39	46	53	60	69	76	83
$\odot$	0	$\odot$	19	26	34	42	50	5$\odot$	69	86	94	$\odot$2
Δ	0	Δ	1$\odot$	29	38	47	56	65	74	83	92	$\odot$0
10	0	10	20	30	40	50	60	70	80	90	$\odot$0	Δ0

[7] I wonder if they’ve done it.

June 18 – think in SEGM

Sam comes early. I give out copies of the first part of this document. He writes after the last footnote: “I did it.” When I ask where it is he blushes and says he left his math notebook home. So I add to his text “but I forgot to bring it.”

When the others arrive it’s clear they have done their homework too, though they haven’t brought theirs either.

We play Buzz. Eva and Grace know the game. They immediately begin to play – in SEGM:

“1, 2, buzz, 4, 5, buzz, 7, 8, buzz, $\odot$, Δ, buzz, table 1, table 2, buzz”

Eva sees that it’s easy. She and Grace are alternating the buzzes, but, more importantly, the last digit tells the whole story.

I ask “Which numbers are the easy ones in our ordinary language?”

“Just two and five.”

“How about in SEGM?”

All look at the multiplication table I’ve provided. Two, three, four and six are easy. They think $\odot$ and Δ will be hard.

It was late June and warm and that’s all we did.

June 8, 2012 – SEGM redux

The four originators have been clamoring for more SEGM for two years. I’ve been reluctant, since Sol wasn’t in the first group and I didn’t want to leave him out.

But now I’ve promised fractions and decimals. He can catch up in advance. It’s SSEGM from now on.

Watch this space for a transcript of the recording Carrie made.