History of Mathematics
Homework 6

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These exercises provide a chance to explore (in modern terms) some of the ways the Italians and then Lagrange worked on cubic equations.

1. Find all three roots of this cubic polynomial (page 231 in our text):

$$c(x) = x^3 - 15x - 4. \tag{1}$$

   You don't need the formula for solving cubics. Since $x = 4$ is one root, $(x - 4)$ is a factor of $c(x)$. The other factor is a quadratic polynomial so you can use the quadratic formula to find its roots.

   Sketch the graph of $c(x)$, showing the roots, the local maxima and minima and the inflection point. (This is an easy calculus 1 exercise.) You can use pencil and paper and scan the result, or, if you're ambitious, do it in LaTeX with tikz/pgfplots.

2. Show that

$$\sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}} = 4 \tag{2}$$

   Hint: Calculate $(2 + i)^3$.

3. Every complex number has three cube roots – when you know one you get the other two by multiplying it by the other two cube roots of 1. So, for example, the three cube roots of $2 + 11i$ are

$$2 + i, \frac{-1 + \sqrt{3}}{2}(2 + i) \text{ and } \frac{-1 - \sqrt{3}}{2}(2 + i).$$

   Using all three cube roots in each term on the left in Equation 2 would seem to give nine answers. Show that there are really only three and that they are the three roots of Equation 1 you found in the first problem.
   [1]

4. Do Exercise 5.4 (page 219) in our text.

[1] I haven't checked this. I'm pretty sure it's right.

Here is the LaTeX source for this document. You can cut it from the pdf and use it to start your answers. I used the `\jobname` macro for the source file name, so you can call your file by any name you like.

```
% Math 370 hw5 Spring 2014
%
\documentclass{article}
\pagestyle{empty}

\usepackage{amsmath}
\usepackage{amsthm}
\usepackage{hyperref}
\usepackage{graphicx}
\usepackage{verbatim}

%% create an environment for theorems
\newtheorem*{thm}{Theorem}

\newcommand{\coursehome}
{http://www.cs.umb.edu/~eb/370}

\title{History of Mathematics \\
Homework 6
}
\author{Ethan Bolker}

\begin{document}

\maketitle

These exercises provide a chance to explore (in modern terms) some of
the ways the Italians and then Lagrange worked on cubic equations.

\begin{enumerate}

\item Find all three roots of this cubic polynomial (page 231 in our text):
%
\begin{equation}\label{eq:cubic}
c(x) = x^3 - 15x - 4 .
\end{equation}

You don't need the formula for solving cubics. Since
$x=4$ is one root, $(x-4)$ is a factor of $c(x)$. The other
factor is a quadratic polynomial so you can use the quadratic formula
to find its roots.
```

```
Sketch the graph of $c(x)$, showing the roots, the local maxima and
minima and the inflection point. (This is an easy calculus 1
exercise.) You can use pencil and paper and scan the result,
or, if you're ambitious, do it in \LaTeX{} with tikz/pgfplots.


\item Show that
%
\begin{equation}\label{eq:weird}
\sqrt[3]{ 2 + \sqrt{-121}} +
\sqrt[3]{ 2 - \sqrt{-121}} = 4
\end{equation}

Hint: Calculate $(2+i)^3$.

\item Every complex number has three cube roots -- when you know one
  you get the other two by multiplying it by the other two cube roots
  of $1$. So, for example, the three cube roots of $2 + 11i$ are
\begin{equation*}
2+i, \frac{-1 + \sqrt{3}}{2}(2+i) \text{ and }
\frac{-1 - \sqrt{3}}{2}(2+i).
\end{equation*}

Using all three cube roots in each term
on the left in Equation~\ref{eq:weird}
  would seem to give nine answers. Show that there are really
  only three and that they are  the three roots of
  Equation~\ref{eq:cubic} you found in the first problem.%

\footnote{I haven't checked this. I'm pretty sure it's right.}

\item Do Exercise 5.4 (page 219) in our text.

\end{enumerate}

\newpage

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pdf and use it to start your answers. I used the \verb!\jobname! macro
for the source file name, so you can call your file by any name you like.

\verbatiminput{\jobname}

\end{document}
```