Relative and Absolute Change Percentages

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Plan

Use the credit card solicitation data to address the question of measuring change.

- Subtraction comes naturally.
- Relative change is more informative, even though division is less natural.
- Units for absolute change are units you started with.
- Relative change is *dimensionless* – the units cancel.
- Percent means, literally, divide by 100. Why do we do that?
- Computing relative change in one step (without computing the absolute change first). That difficult-to-understand 1.
- Change when a quantity decreases.

Lecture notes

We spent almost the whole class on three numbers from the chart – the last line, which says

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All issuers First half 2006:986.0m First half 2007:1.4b 41.5
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where the 41.5 is in an icon that suggests an up-arrow.[1]

We worked through this simple example in much more detail than might seem necessary in order to extract some general principles from the discussion.[2]

We start with the question

> How might you describe the change in total direct mail solicitations from 2006 to 2007?

The first natural answer is that to compare two numbers you *subtract* one from the other. In this case we know we want to subtract the first number from the second, the smaller from the larger. But those aren't good ways to remember the order. The newspaper might have listed the 2007 number first. And sometimes the new value will be smaller than the old one. A better way to think is:

$$\text{change} = \text{value now} - \text{value then}$$

or (when time isn't involved)

$$\text{change} = \text{new value} - \text{reference value}.$$

In this example we want to compute the change in the number of mailings:

$$\begin{aligned} &1.4 \text{ billion mailings} - 986.0 \text{ million mailings} \\ &\quad = 1,400 \text{ million mailings} - 986 \text{ million mailings} \\ &\quad = 414 \text{ million mailings} \end{aligned}$$

(We convert the billions to millions to make the numbers we care about easier to deal with. That way we don't have to write out all the zeroes and line up the columns to do the subtraction.)

Now that we have the difference what does it tell us? Is 414 million a large number? Maybe, maybe not. Here's an analogous question: is an increase of $1000 in annual income a big increase not? The answer is clearly "yes" if you're a student working part time to put yourself through school and you

[1] If this were a graphic about housing it might suggest a house.

[2] Learning new principles is easier in an easy familiar context than in a confusing new one.

currently make about \$10,000 per year. That \$1,000 is a ten percent increase. But if you're a CEO making a million dollars a year the extra \$1,000 doesn't mean much to you at all. It's an increase of just a tenth of a percent.

The previous paragraph suggests that to think about the meaning of a change in the value of some number it helps to compare the change to the original value – and the way to do that is by dividing. In our example that gives us

$$\frac{414 \text{ million mailings}}{986.0 \text{ million mailings}} = 0.419878296 \approx 0.420$$

The calculator gave us all the digits in 0.419878296 but most of those digits are meaningless. Since there are only three significant digits in the numbers we started with we can count on only three in any arithmetic we do. So we round to 0.420.

The number 0.420 *seems* to have two extra zeroes: we were taught in elementary school that 0.420 and .42 are the same. But those extra zeroes aren't really extra. The first one, before the decimal point, doesn't change the meaning. But you should get into the habit of writing it. It helps remind the reader of the decimal point, which can get lost when it starts a number all by itself. The zero after the 2 really does matter, because it tells you something about how accurate the number is. 0.42 will result when you round off anything between 0.415 and 0.425. But you can get 0.420 only by rounding numbers between 0.4195 and 0.4205.

We're almost there. Most people are uncomfortable with numbers between 0 and 1, like 0.420. We're happier with numbers between 0 and 100. Somehow they are friendlier, less frightening. So we have developed the habit of converting decimal fractions to percentages. The "%" sign we're familiar with and the word "percent" mean, literally (in the Latin from which they come) "divide by 100." So

$$42.0\% = \frac{42.0}{100} = 0.420$$

(Note: another good habit to develop is to write fractions with horizontal rather than diagonal fraction bars:

$$\frac{42.0}{100} \text{ instead of } 42.0/100$$

When you have complicated fractions or many of them the diagonals can be very confusing. Paper is cheap so use as much as you need to make your meaning clear.)

We've just computed a 42% increase in direct mail solicitations from 2006 to 2007. But we didn't actually have to do the calculation, since the Globe did it for us. The 41.5 in the up-arrow icon on the chart is just the percentage increase. Newspaper stories often do these kinds of calculations for their readers – in this case computing the relative change rather than simply leaving us to think our own way about the change from 2006 to 2007. From our perspective in this course that's a good thing. We can use that *redundancy* to check their arithmetic and our understanding.

Why then does the chart show a 41.5% increase when we found 42.0%. Is the Globe wrong? Did we make a mistake? No – both answers are right. The difference between our value and the Globe's is too large to be the result of simple roundoff error, but it may be the result of more subtle roundoff error. The 2006 and 2007 figures of 1.4 billion and 986.0 million mailings are clearly themselves rounded off from some other figures. We think that the Globe computed the percentage increase using the values *before* rounding them off for reporting. That is in fact the correct thing to do, and could well account for the difference of a half a percent between their answer and ours.[3][4]

The half a percent difference between our calculation of the percent increase and what the Globe reports does not change our understanding of what the data say. If you were telling a friend about this news story you could say "Did you know that this year credit card companies sent 40 percent more mailings to poor people than they did last year?" – rounding off even further in speech than on paper. And that would not be a distortion.

At the risk of encouraging you to mark parts of these notes with a highlighter instead of reading them, we'll summarize what we've done so far:

$$\textit{absolute}\text{ change} = \text{new value} - \text{reference value}$$

$$\textit{relative}\text{ change} = \frac{\text{new value} - \text{reference value}}{\text{reference value}}$$

and

[3] We don't understand why the Globe reports the 2007 figure of 1.4 billion with two significant digits and the 2006 figure of 986.0 million with four (the zero after the decimal point counts). Perhaps that's how the data came to them. Perhaps they didn't really understand the importance of the distinction.

[4] We should provide a numerical example here showing that rounding before and after computing can make a difference. Perhaps imagine what the 1.4 and 986.0 might have been before rounding to lead to the Globe's 41.5%.

$$
\begin{aligned}
&\text{relative } \textit{percent} \text{ change} \\
&\quad = \text{ relative change} \times 100 \\
&\quad = \frac{\text{new value} - \text{reference value}}{\text{reference value}} \times 100
\end{aligned}
$$

If you count the number of keys on your calculator you need to press to compute the relative change the result is about 15. There's a faster way. Don't bother computing the absolute change first and then dividing. Just divide (rather than subtract) right from the start:

$$
\begin{aligned}
\frac{\text{new value}}{\text{reference value}} &= \frac{1,400 \text{ million mailings}}{986.0 \text{ million mailings}} \\
&= 1.419878296 \\
&\approx 1.420
\end{aligned}
$$

Then just read off the 42.0% increase by ignoring the 1. The cost is about 9 calculator keystrokes, so only about 2/3 the work of doing it the long way.

The disadvantage to the new method is that it's unfamiliar, and perhaps more than a little confusing. What does it mean to "ignore the 1"? Why is that OK?

Stay tuned. We'll find out more in the next lecture.

The class ended with this puzzle:

> Your boss says times are hard and everyone must take a 10% pay cut. Then (before the next payday) he says things have gotten better so everyone gets a 10% pay raise and we're all even.

Are you even? Half the class thought so; some of you were skeptical. The skeptics were right. The quickest way to see what's going on is to start with some fixed dollar amount – say $10 per hour, since 10 is a convenient number for this problem. Then after a 10% cut you're making $9/hour. The 10% pay raise adds $0.9, or 90 cents, so your pay rate after the cut and the raise is just $9.90, not the $10 you started with. Sneaky boss, trying to bank on the fact that you don't understand percentages.

Everyone was satisfied with this explanation – the half who'd guessed wrong won't make that mistake again.

Then I asked

What if the 10% raise came first, then the 10% cut?

The surprising answer is that the boss wins again: the \$10 hourly rate first grows to \$11. Then the 10% pay cut subtracts \$1.10, leaving you with the same \$9.90!

Computing from a \$10 starting rate is a good safe way to solve the problem. We'll see another way in class next time after we've worked harder at understanding the mystery that tells you to "ignore the 1."