Relative and Absolute Change Percentages

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Plan

Continue the work started last time on percent change

- Computing relative change in one step (without computing the absolute change first). That difficult-to-understand 1.
- Computing total cost as price * (1 + taxRate).
- Relative change when a quantity decreases.
- Successive discounts.
- Percentage points.

Lecture notes

We talked about the homework. Many (about 70%) felt that it was a lot of homework. We noted that it is reasonable to expect to do 6 hours per week of work outside the class for a 3-credit class. Since no one admitted to spending more than 6 hours on the homework, and only a few people spent more than 4 hours, it appears that the homework was reasonable.

We started the discussion of relative change with the following question:

> Filene's Basement had an ad in today's newspaper announcing a sale with 40% to 80% off in discounts. Suppose an item costs \$117 and the discount is 40%. What is the discounted cost of the item?

One student suggested the following approach: divide 117 by 10, then multiply the result by 4. This is a way to do the problem in your head, so it makes sense if you don't have a calculator or piece of paper handy. Here's what we get:

$$\frac{117}{10} = 11.7 \text{ and } 11.7 \times 4 = 46.8.$$

Therefore, our *discount* is \$46.80 and our discounted price is

$$\$117.00 - \$46.80 = \$70.20.$$

Another student had a different approach:

$$117 \times 0.40 = 46.8 \text{ and then subtract: } 117 - 46.80 = 70.20.$$

Both these methods first figure out the 40% discount, then subtract that amount from the original price. This is how most people work out the answer. But there's a clever one step method that works too, and is worth learning.

Another student suggested the following:

$$117 \times 0.60 = 70.20.$$

Why do we get the same answer as the above methods? Here is her reasoning: If we have a 40% discount, that means we pay

$$100\% \text{ the original price} - 40\% \text{ of the original price} = 60\% \text{ of the original price.}$$

To summarize: each approach is correct, in that it gives the right answer. The first and second use what you know about discounts; the third approach is the most efficient.[1]

Remember that in the last class we talked about different ways to compute the relative percentage change. We said that you could look at the fraction

$$\frac{\text{new value}}{\text{reference value}}$$

[1] You can combine the trick of the first with the method of the third to to the third in your head: 10% of \$117 is \$11.70. Multiply that by 6 to get $\$66 + \$4.20 = \$70.20$.

to calculate the relative change. That's essentially what the third student's approach does. We know that

$$\frac{\text{new cost}}{117} = 0.60$$

and if we rewrite this equation, we obtain

$$\text{new cost} = 117 \times 0.60 = 70.20.$$

The next question was:

> Because you are a frequent shopper at Filene's Basement, you have a coupon that gives you an additional 10% off any purchase. How can you calculate the 40% and the 10% discounts?

While it is tempting to add the percentages, that doesn't work in this case. Why not? The 40% discount is calculated on one price (the original price) while the 10% discount is calculated on a *different* price, the discounted price. So we need to take the discounts successively. Remember that a 40% discount means that our new price is 60% of the original price. So a 10% discount off of the discounted price would be the same as paying 90% of the discounted price. To put this all together, we have:

$$(117.00 \times 0.60) \times 0.90 = 70.20 \times 0.90 = 63.18.$$

In this example, our total cash discount is \$117.00 − \$63.18 = \$53.82. That means that our savings (in other words, our total percentage discount) is

$$\frac{53.82}{117.00} = 0.46 \text{ or } 46\%.$$

How do we know whether to multiply or divide, or how to set up the fraction? Remember that a percentage is another way to talk about a fraction of a whole. The whole, or original, is \$117, so that is what we put in the denominator of the fraction.

What if we took the 10% discount first, then the 40% discount. Would that change our final cost? Let's check:

$$(117 \times 0.90) \times 0.60 = 63.18.$$

So the order of discounts doesn't matter. You can re-write the above equation to group the percentages first. We would have:

$$117 \times (0.90 \times 0.60) = 117 \times 0.54 = 63.18.$$

By doing this, we have another way to see that we are paying 54% of the original price, or saving 46% of the original price. As we noted earlier, this is quite different from what we would get if we added the percentages.

The next topic was sales tax. In Massachusetts, sales tax on most items is 5%. If we pay full price for our $117.00 item, what is the sales tax?

Our student who likes to do these problems in his head had the following suggestion: Multiply the 117 by 0.10, then cut that value in half to get the sales tax. Here's the calculation:

$$117 \times 0.10 = 11.70 \text{ and } \frac{1}{2}(11.70) = 5.55.$$

We add this to the original cost to get a final cost of

$$\$117.00 + \$5.55 = \$122.55.$$

This is a good way to estimate or to do the calculation in your head. Here is another approach: Multiply 117 by 1.05. The calculation is:

$$117.00 \times 1.05 = 122.55.$$

Note that this takes us immediately to the final answer. Why does this work? It uses the following:

$$\begin{aligned} \text{final price} &= 100\% \text{ of the ticket price} + 5\% \text{ of the ticket price} \\ &= 105\% \text{ of the ticket price.} \end{aligned}$$

Because we are taking the percentages of the same amount (the ticket price) we can add the percentages. That is, the final price is 105% of the ticket price. This is the same principle that we used in calculating our discounted price, only in that case we subtracted (because it was a discount).

Another question: If you have a 10% discount, should the store calculate the discount before calculating the sales tax or after? Try it - you'll see that you get the same final price either way. However, the state has an interest in how this calculation is made, as the tax is different in the two cases.

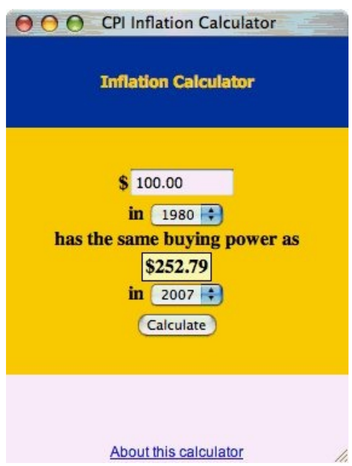
Important point: in all of these cases, we are really doing successive multiplication. That is a good approach to finding the answer.

Consumer Price Index

We spent the rest of the class time talking about the Consumer Price Index, which measures inflation. The class had a general sense that inflation has to do with rising prices and costs. A definition found on the web started out with "a sustained increase in the general level of prices...."

A more specific question: what does it mean to say that inflation if 4%? One student's response: if a candy bar cost \$1.00 last year, it will cost \$1.04 this year. This would be a literal interpretation of all prices increasing by 4%. More likely, the candy bar manufacturer would keep the price at \$1.00 and would make the candy bar a bit smaller.

One way to determine the inflation rate from previous years is to look at the Consumer Price Index. This is calculated by the U.S. Bureau of Labor Statistics and measures how the prices of a representative basket of goods and services changes. We can look at an inflation calculator on the BLS website and, for example, compare 1980 with 2007. Here is what the calculator tells us:



When we edit these notes we should do two calculations - the first for a shorter number of years, so that the inflation rate is less than 100%. Then do this one.

How do we interpret this? Generally speaking, this means that if we paid \$100 for an item back in 1980, it would cost about \$253 today. That means that prices (on average) have increased by a factor of

$$\frac{253}{100} = 2.53$$

That means they are more than two and a half times what they were. To express the increase as a percentage we observe that

$$\begin{aligned} \text{2007 prices} &= 2.53 \times \text{1980 prices} \\ &= (1.00 + 1.53) \times \text{1980 prices} \\ &= \text{1980 prices} + 1.53 \times \text{1980 prices} \end{aligned}$$

so prices *increased* by a factor of 1.53, or 153% in the 27 year time period. In short, we subtract 1 from the 2.53 since the 1 represents 100% of the 1980 value of the dollar and we are interested in just the *increase*.

An easier example is to see how the buying power of $100 in 1980 increases by 1981. The inflation calculator says that $110.32 has the same buying power in 1981 as $100 had in 1980. The inflation rate is the percentage increase here, about 10.3%. In comparison, the inflation rate from 2006 to 2007 is about 3.3%.

A good question to think about: why do we put $100 into the inflation calculator? What would happen if you just put in $1.00? Try it and see if you get the same information about the inflation rate. (This is a question on the homework.)

Last question to think about: Suppose you received a 10% raise from 2006 to 2007. Can you buy 10% more in 2007 than you could have bought in 2006? Not if you take inflation into account. Remember that the inflation rate in this time period was about 3%. That means that you can only buy about 7% more in 2007 than you could have bought in 2006.