

# Percentages, continued

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## Plan

- Finish discussion about consumer price index
- Review of calculations using percentages
- Percentage points

## Lecture notes

In the last class, we finished with the following question:

If your salary increased by 10% between 2006 and 2007, does your buying power also increase by 10%?

We figured out last time that the inflation rate of about 3% cuts into our buying power, and we estimated that our buying power effectively only increases by about 7%. We look at this again, first with a slightly simpler example.

Suppose your salary in 2006 was \$40,000 and your contract stated that you would receive a cost of living raise equal to the inflation rate. From 2006 to 2007, the inflation rate was 3.3%. Therefore, your salary would increase to \$41,320, since

$$\$40,000 + 0.033 \times (\$40,000) = \$40,000 + \$1,320 = \$41,320$$

or, computing the same answer in one step

$$\$40,000 \times 1.033 = \$41,320.$$

This seems like a good deal. You are taking home more money in your paycheck, and your overall salary has increased. However, if we take inflation into account then the situation is not quite as good as it seems. Because of inflation, you would need all of the \$41,320 in 2007 to have the same buying power as you had in 2006, when your salary was \$40,000. Since your raise is equal to the inflation rate, your buying power has not changed at all. Effectively, your salary has *not* increased.

Of course you were clever enough to know this, so you negotiated a 10% raise from 2006 to 2007 so that you would actually be earning more after taking inflation into account. Then your 2007 salary would be \$44,000.<sup>1</sup> How much has your buying power increased? The inflation rate was 3.3%, so it's reasonable to estimate an increase of  $10\% - 3.3\% = 6.7\%$  in buying power. That's only an estimate, though, since we know you can't reliably subtract percentages this way. To compute the answer carefully, remember that we just figured out that you would need \$41,320 in 2007 to maintain the buying power you had in 2006. To see how your buying power has *effectively* changed, calculate the relative change:

$$\text{relative change} = \frac{\text{new value} - \text{reference value}}{\text{reference value}} = \frac{44,000 - 41,320}{41,320} = 0.0646.$$

That is, your buying power has really only increased by 6.46%. We had estimated a 6.7% increase in buying power, so we were not too far off.

As another example, we look at the federal minimum wage. The federal minimum wage is the basic minimum wage that most employees are entitled to. In 1991, the federal minimum wage was \$4.25 per hour. The minimum wage increased slowly, then stayed constant for over five years. In July 2007, it increased to \$5.85 per hour. If we just look at the numbers, it appears that the minimum wage has increased since 1991. If we take inflation into account, however, we get a different perspective. The buying power of the dollar has changed since 1991. In 2007, you would need \$6.50 to have the same buying power that \$4.25 had in 1991. Since the minimum wage has only increased to \$5.85, employees earning that minimum wage do not, in general, have the same buying power that employees earning the minimum wage had in 1991. Therefore, the minimum wage has effectively decreased.

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<sup>1</sup>You should be able to calculate a 10% increase in your head:  $40,000 \times 1.10 = \$44,000$ .

## Review of percentage calculations

As a reminder, here is how we calculate percentages and percentage change.

When we calculate a percentage, we are calculating a fraction. As we said earlier, the word “per” means “divide”. Let this guide you in setting up the fraction. Another way to think of a percentage is to think of it as a proportion. To determine how many students at UMass Boston are women, for example, we would set up the fraction

$$\frac{\text{number of women students}}{\text{total number of students}}.$$

Say more about this? General definition?

When we compare two numbers, one very powerful way to make a comparison is to calculate a percentage change. We first calculate a relative change, which shows how the new value has increased or decreased relative to the reference value. To make this into a percentage, we multiply by 100. Here are the basic formulas we are using:

$$\begin{aligned} \text{relative change} &= \frac{\text{new value} - \text{reference value}}{\text{reference value}} \\ &= \frac{\text{new value}}{\text{reference value}} - 1. \end{aligned}$$

To turn the relative changes into percentages, adjust the calculations as follows:

$$\begin{aligned} \text{relative } \textit{percent} \text{ change} & \\ &= \text{relative change} \times 100 \\ &= \frac{\text{new value} - \text{reference value}}{\text{reference value}} \times 100 \end{aligned}$$

By rearranging the last equation, you can see another way to calculate relative change:

$$\begin{aligned} \text{relative } \textit{percent} \text{ change} & \\ &= \text{relative change} \times 100 \\ &= \frac{\text{new value}}{\text{reference value}} \times 100 - 100 \end{aligned}$$

Now for some examples. First, we calculate a percentage. The fall enrollment at UMass Boston, according a report just released from the Office of Institutional Research, is 13,220 students. In the report, it notes that 3,261 of these students are graduate students. How can we express this as a percentage? We can form a proportion:

$$\frac{\text{number of graduate students}}{\text{total number of students}} = \frac{3,261}{13,220} = 0.246.$$

In the report, we might write

Approximately 25% of the UMass Boston student enrollment consists of graduate students.

Note that this tells us immediately that 75% (or 100% - 25%) of the UMass Boston student enrollment consists of undergraduate students.

The enrollment report also noted that UMass Boston saw a 9% increase in enrollment this fall, compared to the enrollment totals in fall 2006. We know that the enrollment this fall is 13,220. Can we find the enrollment in fall 2006?

We could write

$$\begin{aligned} \text{Fall 2007 enrollment} &= 100\% \text{ of Fall 2006 enrollment} \\ &\quad + 9\% \text{ of Fall 2006 enrollment} \\ &= 109\% \text{ of Fall 2006 enrollment.} \end{aligned}$$

Where does the 109% come from? We have added 100% to 9%. This is a reasonable calculation to make, since the 100% is taken in reference to the fall 2006 enrollment and the 9% is taken in reference to the same base number.

To finish the calculation, we put in what we know (Fall 2007 enrollment is 13,220) and we write "Fall 2006 enrollment" as  $x$ , both to save writing time and to make the equation easier to understand. We obtain

$$\begin{aligned} 13,220 &= 109\% \text{ of } x \\ &= 1.09x \end{aligned}$$

and to solve the equation, we divide both sides by 1.09 to get  $x$  by itself:

$$\frac{13,220}{1.09} = \frac{1.09x}{1.09}, \text{ or } x = 12128.44.$$

Now we interpret the answer. The fall 2006 enrollment was about 12,128 students (note that we drop the 0.44 since it doesn't make sense to have 0.44 students).

You can always double check your answer. Is it true that 13,220 is 9% greater than 12,128? Do the math:

$$\begin{aligned}12,128 + 0.09 \times 12,128 &= 12,128 + 1,092 \\ &= 3,220.\end{aligned}$$

Now we try another example, with a slightly different approach. Suppose you received a 10% raise this year and your new salary is \$45,000. What was your salary last year? We can make the following comparison:

$$\frac{\text{this year's salary}}{\text{last year's salary}} = 1.10.$$

If we let  $x$  represent last year's salary, then we can solve the equation for  $x$ :

$$\begin{aligned}\frac{45,000}{x} &= 1.10 \\ 45,000 &= 1.10x \\ \frac{45,000}{1.10} &= x \\ 40,909 &= x.\end{aligned}$$

In other words, your salary last year was about \$40,909.

## Percentage Points

We look at one more comparison using percentages. If the two numbers you are comparing are *already* percentages, then you can compare them using the language of percentage points. The arithmetic is simple here: just subtract the two numbers. However, your answer will only make sense if you use *percentage points* when you write it.

As an example, we can compare interest rates. The Federal Reserve Board sets interest rates (in part to try to control inflation, but that's a complicated story), which in turn determine interest rates for savings accounts and for loans such as mortgages. If an interest rate rises from 6.75% to 7.25%, how can we describe that change? We can say the following:

The interest rate has risen 0.5 percentage points.

The 0.5 comes from simple subtraction ( $7.25 - 6.75$ ) and we can do that subtraction as long as we express the answer using percentage points.

Challenge question: What if a student had written

The interest rate has risen 0.5%

to try to describe the increase in the interest rate from 6.75% to 7.25%. Why would this be incorrect? We'll review this next time and talk a bit more about percentage points.