# Manipulating units

Ethan D. Bolker
Maura B. Mast

September 20, 2007

## Plan

We did plan this class, but never wrote the plan into the notes.

A standard textbook covers each topic in its own place. The author orders the topics in a way that makes intellectual sense, in order to build a coherent experience for the student. Real life is different. You can be called upon to solve any problem at any time, whether or not you've "covered the material" in some previous chapter of your life. Our experiment teaching quantitative reasoning is somewhere in between. We have a general idea of the topics we want to explore, but the detailed schedule is driven not by our ideas but by the needs of the class and the problems suggested by the days news. So it's somewhere between a formal syllabus and real life.

This lecture/class is an example. We discussed "rate $\times$ time = distance" in the class, but the written notes are now in the previous lecture, where they "belong." The discussion of inflation here belongs in an earlier section, but served in class as a good review. The discussion of billions was suggested by newspaper article, and so moved Fermi problems like heartbeat estimation earlier in the syllabus.

## Lecture notes

### Percentage Increase Redux

Here is one of the homework problems that caused trouble:

> Use the on line Inflation Calculator to compute the inflation rate (as a percent) from 2000 to 2003 and the rate from 2003 to 2007. Use these two rates to compute the inflation rate from 2000 to 2007. **Do this with your calculator, not with the inflation calculator from the web page.** Explain why what you do will work. Then check your result using the web calculator.

When you enter that data the calculator tells you that \$100 in 2000 has the same buying power as \$106.85 in 2003. That corresponds to 6.85% inflation for the three year period. The calculator says that \$100 in 2003 has the same buying power as \$113.21 in 2007. That corresponds to an inflation rate of 13.21% .

The easiest way to combine the inflation rates is to understand the "multiply by 1+rate" idea we've worked with regularly. In this case we compute

$$1.0685 \times 1.1321 = 1.20964885 \approx 1.2096$$

so the inflation rate from 2000 to 2007 was 20.96%. The on line calculator confirms that answer.

Many people think you can combine the these rates by adding them. If you did you would conclude that the inflation rate from 2000 to 2007 was 6.85% + 13.21% = 20.06%. That's wrong. It's too small. It does not take into account the fact that the second round of inflation should start with the already inflated 2003 dollars. If you need to remember a safe general rule

*Never add (or subtract) percentages.*

Well, hardly ever. Sometimes (rarely) it's OK. When it is, the change in the value of a percent is usually expressed in *percentage points.*

## Billions of phone calls?

On September 19, 2007 *The Boston Globe* reported that

> Mike McConnell, director of national intelligence, could not say how many Americans' telephone conversations have been overheard because of US wiretaps on foreign phone lines.
>
> "I don't have the exact number ... considering there are billions of transactions every day," McConnell told the House Judiciary Committee at a hearing on the law governing federal surveillance of phone calls and e-mails.

When I read this I was skeptical. So we set out in class to see if this was a reasonable number. The population of the United States is 300,000,000. Who makes calls to foreign countries? People with friends and relatives abroad. Many businesses. People calling customer service centers may well be making overseas calls, since many of those centers are outsourced. Clearly the number of people is hard to estimate. But it seems safe to say that it's no more than 10% of the population.

Now we can estimate. The smallest "billions of calls" could be is about 2 billion. If 10% of the people in the country make 2 billion calls then the average number of calls each one makes is

$$\frac{2,000,000,000 \text{ calls}}{30,000,000 \text{ people}} \approx 70 \ \frac{\text{calls}}{\text{person}}$$

It seems unlikely that one person in every 10 in the United States averages 70 foreign telephone calls each day. We suspect that McConnell does not actually know the number of calls; he's just trying to explain why his job is so hard.

## How Many Heartbeats in a Lifetime

This is another question that's worth an order of magnitude estimate even though an exact answer is impossible. We started with a poll:

What do you think the answer will be?

- thousands: 0
- millions: 3
- billions: 5
- trillions [1] : 2
- no opinion: the rest

[1] When I'd made the list but before taking the poll someone asked "what comes after trillions?" That led to an interesting digression about zillions – a generic name for an unspecified very big number. (My grandson has some thoughts about zillions I'll put here soon.) Referring back to the previous section, perhaps McConnell should have said there were zillions of calls instead of making up a number. Hee'd have been more accurate (but perhaps less convincing). The metric system handles large numbers better – we'll see this soon.

To answer the question we need to know (approximately) the length of a lifetime and the value of a heartbeat. We measure lifetimes in years. Let's be optimistic and assume a long healthy life: 90 years. To find out how fast hearts beat we took a minute off to take our pulses. 70 $\frac{\text{beats}}{\text{second}}$ was a fair average for the class. To convert from $\frac{\text{beats}}{\text{minute}}$ to $\frac{\text{beats}}{\text{lifetime}}$ we chained together the time unit conversions, and then approximated all the numbers so we could do the arithmetic without a calculator:

$$
\begin{aligned}
&70\frac{\text{beats}}{\text{minute}} \times 60\ \frac{\text{minutes}}{\text{hour}} \times 24\ \frac{\text{hours}}{\text{day}} \times 365\ \frac{\text{days}}{\text{year}} \times 90\ \frac{\text{years}}{\text{lifetime}} \\
=&\ 70 \times 60 \times 24 \times 365 \times 90\ \frac{\text{beats}}{\text{lifetime}} \\
\approx&\ 70 \times 50 \times 20 \times 400 \times 100\ \frac{\text{beats}}{\text{lifetime}} \\
=&\ (70 \cdot 400) \times (50 \cdot 20) \times 100\ \frac{\text{beats}}{\text{lifetime}} \\
=&\ 28000 \times 1000 \times 100\ \frac{\text{beats}}{\text{lifetime}} \\
=&\ 28 \times 10000000\ \frac{\text{beats}}{\text{lifetime}} \\
\approx&\ 3 \text{ billion}\ \frac{\text{beats}}{\text{lifetime}}
\end{aligned}
$$

So the guesses in the billions were right. That fact is independent of the assumptions we made. If we worked out the actual arithmetic we'd have found 3,311,280,000. But those extra significant digits are meaningless given the approximations we started with for pulse rate and the length of a lifetime. If either of those approximates were off by ten or twenty percent, so would the answer: still in the low billions.