# Estimation, Order of Magnitude

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## Plan

## Lecture notes

- How many books could I store in my computer?
- Scientific notation and powers of 10
- Metric prefixes

### How many books could I store in my computer?

Today we talked about several topics at once: how do we estimate, how do we solve a problem, and how do we use numbers and units to get to the solution. We also talked about large numbers – numbers with many zeros – and the metric prefixes that are used to describe them.

Our starting point was the question “How many books could I store in my computer?” We don’t expect an exact answer to this question and we don’t expect that you will know every piece of information before you start thinking about the problem. In fact, part of the reason we do this problem is to illustrate the steps that can be used to solve prolems.

We started with a guess. Could we store thousands of books, or millions? Or should it be billions of books, or trillions? About half of the class guessed that we could store thousands or millions of books, and the rest felt it would be more. We’ll see who had the right intuition about this soon.

What do we need to know to solve this problem? Here are the ideas the class came up with:

- How big is the space available on my computer? This is measured in bytes.
- How large is each book (or computer file containing a book)?
- How do we measure a book? Is it in pages, chapters, bytes, characters?

We can answer the first question by looking at information about the computer in front of us. The computers that we are using hold about 232.57 gigabytes. We could use 232 gigabytes, but 200 gigabytes is even easier to use and won't lose us much in our estimation. What is a gigabyte? The metric prefix *giga* represents nine 0's, or a billion. In the context of the problem we're looking at,

`1 gigabyte=1,000,000,000 bytes.`

Now we need to figure out how large a file containing a book would be. We are looking for an average, or representative, size. We estimated that a book should contain about 300 pages (although 150 pages would work, in that our final answer would have the same number of zeros). But how do we convert pages to bytes? Here it helps to know a bit of outside information, namely that one byte is used to represent an individual character in a text document. This information helps, but doesn't immediately get us the answer of how many bytes are in a typical book file. We need to get more information, namely the number of characters on a typical page in a typical book. Note the important approach that we are using here:

`Make an impossible-looking problem manageable by`
`breaking it down into smaller pieces.`

We could count the letters on a page, but that would take a lot of time. Or we could type a page and count the number of letters. Or, we could count the number of letters in one line and multiply that by the number of lines on a page. All of these approaches would work. We chose the last approach and estimated 50 lines on a page and 50 characters in a line. Now we are ready to calculate! Our first step is to calculate how many bytes are in a typical book:

$$300 \text{ pages} \times \frac{50 \text{ lines}}{\text{page}} \times \frac{50 \text{ bytes}}{\text{line}} = 750,000 \frac{\text{bytes}}{\text{book}}.$$

With this information, we can answer the question of how many books we can fit on our computer:

$$200 \text{ gigabytes} \times \frac{1,000,000,000 \text{ bytes}}{1 \text{ gigabyte}} \times \frac{1 \text{ book}}{750,000 \text{ bytes}} = 267,000 \text{ books.}$$

Our conclusion: we could store about 300,000 books on the computers in our classroom. It's quite possible that the actual number is more or less than 300,000, but we are confident that any answer would be in "hundreds of thousands" of books.

As an example to try on your own, you might attempt to determine how many songs you could store on 40 Gigabyte iPod. Start with a list of information you need to know, which in this case would be the average size of a song. A similar question: how many movies could you store on your computer? Once you see the general approach, you can apply it to many other estimation problems.

## Metric prefixes, the Google calculator, scientific notation

Most quantitative reasoning textbooks devote space to a systematic discussion of scientific notation. We were about to teach that and write it up when we realized that it's rarely needed in the kind of everyday numeracy we want our students to practice. So we left most of it out. In this section we'll talk about the little we think is necessary, tied to a discussion of the metric prefixes.

Essentially, we think it's important to be able to *read* numbers written in scientific notation, but not necessarily to be able to *write* them or compute with them – although that is sometimes a useful skill.[1]

Scientific notation is a useful way to write numbers without having to list all of the zeros. If we experiment with the Google calculator, we see that it writes all of the zeros up to a certain point. Try it and see: if you put "1 kilometer in meters" into the Google search engine, it will return

```
1 kilometer = 1000 meters.
```

Google continues to show the zeros up through terameters. If you ask it to find "1 terameter in meters" it will return

[1]As we discovered in class when it helped uncover a mistake on the whiteboard.

```
1 terameter =  1,000,000,000,000 meters.
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That is, 1 terameter is equal to one trillion meters.

But if you input “1 petameter in meters” then Google decides to switch to scientific notation. At this point, there are really too many zeros to keep track of. Even with the commas to help the eye see the blocks of three zeros, it’s just too long to write without the possibility of making an error. Here’s what Google gives when we ask it to find “1 petameter in meters”:

$$1 \text{ petameter} = 1 \times 10^{15} \text{ meters.}$$

So for reading big numbers in Google (or in the display on your calculator) it is helpful to know how to read numbers in scientific notation.

As a matter of curiosity, note what Google does when we ask it to find “1 kilobyte in bytes.” We would expect the answer to be “1000 bytes,” but in fact Google returns

```
1 kilobyte = 1024 bytes.
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While this is not too far off from what we expected, it is a bit surprising. The reason behind this is that computers work with bytes in powers of 2, in a fairly natural way, and that the metric prefixes were traditionally used for powers of 2, not powers of 10. You can calculate some powers of 2 and fairly quickly discover that $2^{1}0 = 1024$.

The beauty of the metric system is that we can use powers of 10 to move from one unit to another. Since multiplying by a power of 10 is fairly easy, this makes it easy to move among the units. In contrast, converting units in the English system means knowing (or looking up on the web) a very different set of conversions: 12 ounces is equivalent to 1 foot; 3 feet is equivalent to 1 yard; 4 quarts is equivalent to 1 gallon, and so on. Even if you could remember the various conversions, it’s usually much more work to multiply by 12, or 4, or 3 than to multiply by 10.

The key to multiplying by 10 is the key to understanding how scientific notation works: every time you multiply or divide by one copy of 10, you move the decimal point over one place. You can memorize the general rule, or you can just try a few simple examples to see the trend. Here’s the general rule:

```
When you multiply a number by a positive power of 10, you move
the decimal place to the right.  The number of spaces the decimal
moves is equal to the power of 10 by which you are multiplying.
```

As an example, verify the following. We have included the decimal point to emphasize its location.

$$\begin{aligned} 1. \times 10^4 &= 10^4 = 10{,}000. \\ 2. \times 10^4 &= 20{,}000. \\ 5.4 \times 10^4 &= 540{,}000 \\ 0.000\ 000\ 5 \times 10^4 &= 0.005 \end{aligned}$$

In all of the above examples, we multiplied by a power of 10 and moved the decimal place to the *right*. This makes sense, because multiplying by 10 means that we make a number bigger.

When we divide a number by a positive power of 10, we make that number smaller. Here's the general rule:

```
When you divide a number by a positive power of 10, you move
the decimal place to the left.  The number of spaces the decimal
moves is equal to the power of 10 by which you are dividing.
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The only twist on this is that we often write division by a power of 10 as multiplication. You can remember the general rule

$$10^{-n} = \frac{1}{10^n}$$

or work out an example to remind yourself of it. Here are some examples:

$$\begin{aligned} 1. \times 10^{-3} &= \frac{1}{10^3} = \frac{1}{1000} = 0.001 \\ 2. \times 10^{-3} &= 0.002 \\ 5.4 \times 10^{-3} &= 0.0054 \\ 0.000\ 000\ 5 \times 10^{-3} &= 0.000\ 000\ 000\ 5. \end{aligned}$$

As one last example, try asking Google to find "1 petabyte in bytes." The answer is

$$1 \text{ petabyte} = 1.12589991 \times 10^{15} \text{ bytes}.$$

This is already an approximation, but we could write it out the long way by moving the decimal place over 15 spaces to the right.

A good reference for the metric prefixes and large numbers is the following webpage: http://www.unc.edu/ rowlett/units/prefixes.html.

## How much does it cost for electricity to keep a 40 watt bulb on all year in your basement?

Here we answer that question without even knowing what a *watt* is.[2]

As with the first question we looked at, we asked for a guess using powers of 10. Would this cost between $1 and $10? Between $10 and $100? Between $100 and $1,000? Or more than $1,000? Everyone thought it would be between $10 and $100 or between $100 and $1,000. We won't find out the exact answer, but we will get an estimate that places it in one of these ranges.

If you look at an electricity bill, you'll notice that the rate is listed in dollars per *kilowatt hours*. We haven't talked about what a watt is, but it doesn't really matter, since we now know that a kilowatt is 1,000 watts. We can estimate the charge per kilowatt hour, or search for it on the web. Let's use $0.10, both because it's fairly close to the actual rate and because it is an easy number to work with.

What other information do we need? We are assuming (to make things smoother) that our bulb does not burn out. In other words, we are just estimating the cost of electricity. We are asking for the cost over one year, and the rate is given in dollars per kilowatt hour, so we will need to know how many hours are in one year. We've calculated this a couple of times already, but an estimate at this point is good enough. Let's say 9000 hours in one year. Now we are ready for the calculation:

$$40 \text{ watts} \times \frac{\$0.10}{\text{kilowatt hours}} \times \frac{1,000 \text{ watts}}{1 \text{ kilowatt}} \times \frac{9,000 \text{ hours}}{1 \text{ year}} = \$36.$$

If we had used a 100 watt bulb, it would cost a bit more than twice as much, or around $90. In both cases, the estimated cost is within the $10 to $100 range.

[2] We will try to avoid all jokes that depend on the phrase "what's what".