Linear functions

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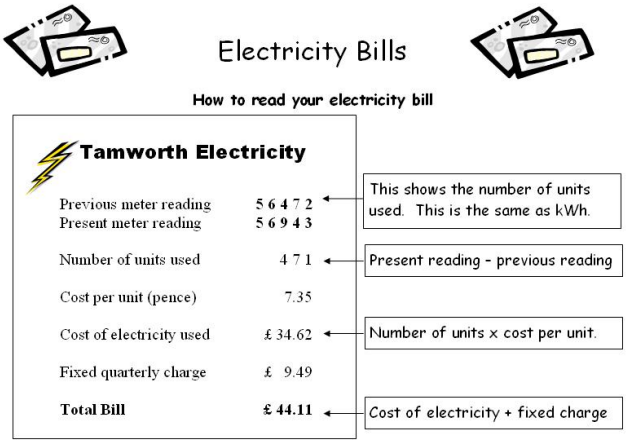
Plan

- The Tamworth Electric Bill
- A linear equation
- Programming in Excel.

Lecture notes

Reading your electric bill

At `http://www.squashedfrogs.co.uk/resources/2005/10/how_to_read_electricity_bills.doc` you will find this sample electric bill:



The bill explains itself. Let's read it from the bottom up. The last line is the total bill, computed as

Cost of electricity + fixed charge

Checking the arithmetic:

$$£44.11 = £34.62 + £9.49$$

The third line from the bottom explains the £34.62:

Number of units × cost per unit

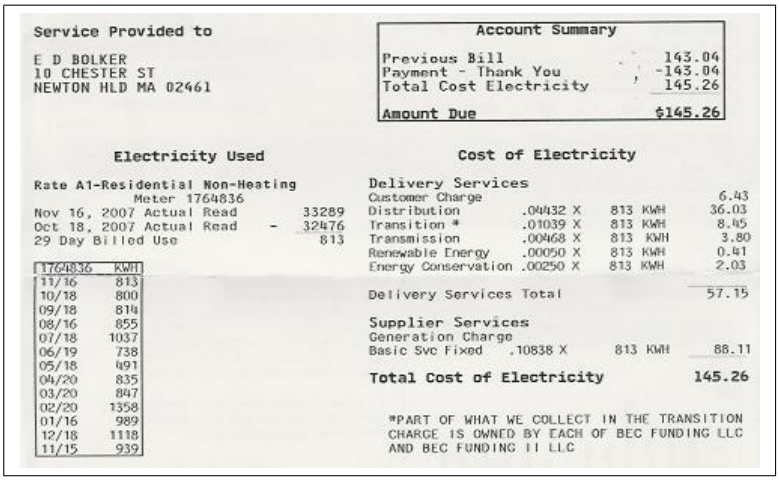
The previous line gives the cost per unit as 7.35 pence. Later on in the document we're told that a unit is just a kilowatt-hour. We know there are 100 pence in a pound, so we write the cost per unit as

$$0.0735 \frac{£}{\text{kwh}}$$

Here's all the arithmetic, with units:[1]

$$£44.11 = 0.0735 \frac{£}{\cancel{\text{kwh}}} \times 471 \cancel{\text{kwh}} + £9.49$$

That English electric bill is easy to read. Here's one that's a little more complex, from NStar, in Boston.



Service Provided to

E D BOLKER
10 CHESTER ST
NEWTON HLD MA 02461

Account Summary

Previous Bill	143.04
Payment - Thank You	-143.04
Total Cost Electricity	145.26
Amount Due	$145.26

Electricity Used

Rate A1-Residential Non-Heating
Meter 1764836

Nov 16, 2007 Actual Read		33289
Oct 18, 2007 Actual Read	-	32476
29 Day Billed Use		813

1764836	KWH
11/16	813
10/18	800
09/18	814
08/16	855
07/18	1037
06/19	738
05/18	491
04/20	835
03/20	847
02/20	1358
01/16	989
12/18	1118
11/15	939

Cost of Electricity

Delivery Services			
Customer Charge			6.43
Distribution	.04432 X	813 KWH	36.03
Transition *	.01039 X	813 KWH	8.45
Transmission	.00468 X	813 KWH	3.80
Renewable Energy	.00050 X	813 KWH	0.41
Energy Conservation	.00250 X	813 KWH	2.03
Delivery Services Total			57.15
Supplier Services			
Generation Charge			
Basic Svc Fixed	.10838 X	813 KWH	88.11
Total Cost of Electricity			145.26

*PART OF WHAT WE COLLECT IN THE TRANSITION CHARGE IS OWNED BY EACH OF BEC FUNDING LLC AND BEC FUNDING II LLC

[1] It's too bad the electric bill talks about "units" instead of just "kwh" since we have another meaning for "units."

We can identify the same two components. The fixed charge is the $6.43 labelled "Customer Charge" on the bill. It's the only contributor to the $145.26 total that does not depend on the amount of electricity I used – in this case, 813 kWH. There are six lines on the bill that do depend on it. They contribute

$$(0.04432 + 0.01039 + 0.00468 + 0.00050 + 0.00250 + 0.10838) \times 813$$

$$\begin{aligned} &= 0.17077 \times 813 \\ &= 138.83601 \end{aligned}$$

to the total bill, which is

$$145.26\$ = 0.17077 \frac{\$}{\cancel{\text{kwh}}} \times 813 \cancel{\text{kwh}} + 6.43\$$$

(If you check the calculation you will discover that the electric company rounded $138.83601 down to $138.83 rather than up to the nearest penny. We should be grateful for small favors.)

Linear equations

So far this semester we've used hardly any algebra explicitly. Now a little bit will come in handy.

Suppose you are working on your household budget and want to estimate electricity cost for some particular month. The monthly $6.43 Customer Charge will stay the same until NStar notifies you of an increase. The amount you pay for the actual electricity you use, 0.17077 $/kWH, can change from month to month, but you have no information about how, so assume it stays the same. Then in a month in which you use E kWH of electricity you can compute your bill B with the formula

$$B\$ = 0.17077 \frac{\$}{\cancel{\text{kwh}}} \times E \cancel{\text{kwh}} + 6.43\$$$

This may look familiar without the units

$$B = 0.17077 \times E + 6.43$$

and perhaps more familiar still if we call the independent variable x instead of E and the dependent variable y instead of B:

$$y = 0.17077x + 6.43$$

Linear equations in Excel

We can use the linear equation for electric bills in Excel to prepare a table of values

	B	C
7	electricity used	total bill
8	(kWH)	(British pounds)
9	0	9.49
10	471	63.95
11	100	16.84
12	355	35.58
13	927	77.62

To build this table we entered a few values rows 9 through 13 in column B. We put the fixed quarterly cost, £ 9.49, in cell B4 and the cost per kWH, 0.0735 £/kWH, in cell B5. Then we put the formula

=$B$5*B9+$B$4

in cell B9 and copied that formula to the rest of column B.

We used the $ trick to keep Excel from changing the references to cells B4 and B5 when we copied the formula from row to row, so the entry in cell B13 is

=*B*5*B13+*B*4

The graph displaying the data in this table is a straight line – that's why the equation is called "linear". The slope tells us how steep the line is, the intercept tells us where it crosses the vertical axis – in this case at the value £ 9.49, the total bill when you use no electricity at all.