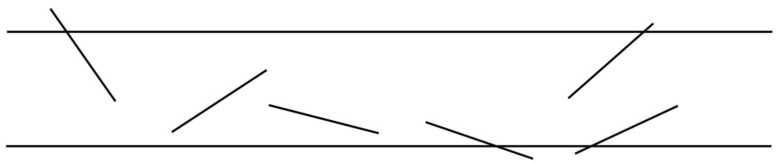
Why π?

Toss thousands of one foot sticks onto stripes one foot apart on the floor. Then, miraculously,

$$\pi \approx \frac{2 \times \text{ number of tosses}}{\text{number of crosses}} \quad !$$

Where does that π come from?

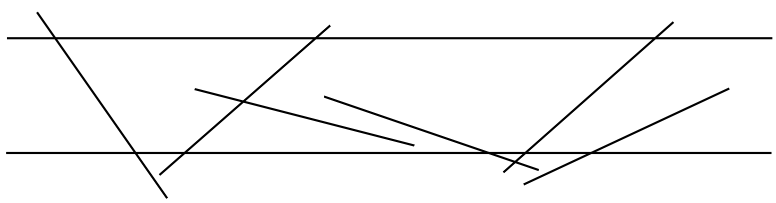
Here's one small experiment, with six sticks (not thousands):



There are four crosses, so

$$\text{average number of crosses} = \frac{\text{number of crosses}}{\text{number of tosses}} = \frac{4}{6} = \frac{2}{3} \quad .$$

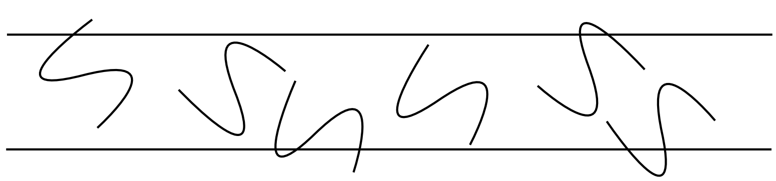
Try the same experiment with sticks twice as long. Some of the sticks that didn't cross before will now, and some that crossed one stripe will now cross two. The picture shows eight. The average number of crosses is $8/6 = 4/3$.



It's a coincidence in this simple example that the average number is exactly twice as large when the sticks are twice as long. Here's what mathematicians can prove in general:

> *In the long run the average number of crosses is proportional to the length of the stick.*

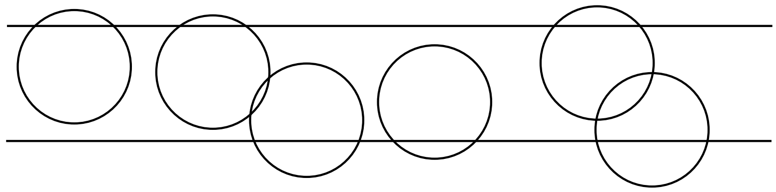
Now suppose that suddenly the sticks turn into snakes! They're just as long as they were before, but they wiggle. Some that used to cross, don't now. Some cross multiple times. The picture might look like this:



The number of crosses is still eight. The average number of crosses is still $4/3$. Again that's a coincidence in this tiny example. In general mathematicians can prove that:

In the long run the average number of crosses depends only on the length of the snake, not on its shape.

Next let the magical circus snakes roll up into circles, with diameter just the distance between the stripes. Here's the new picture:



No matter how many snake circles you toss, each crosses twice so

$$\text{average number of crosses} = \frac{\text{number of crosses}}{\text{number of tosses}} = 2 \quad .$$

Since the stripes are one foot apart, the circular snakes have diameter 1. That means their circumference is π. Since shape doesn't matter, sticks π feet long would cross twice on average too. But the average number of crosses is proportional to the length of the stick, so for thousands of sticks just one foot long ...

$$\text{average number of crosses} = \frac{\text{number of crosses}}{\text{number of tosses}} \approx 2/\pi \quad ,$$

which, with the littlest bit of algebra, says

$$\pi \approx \frac{2 \times \text{ number of tosses}}{\text{number of crosses}} \quad !$$

But ... is this a good way to calculate π?

Well, not very. Suppose you tossed *ten thousand* sticks. Working backwards, you would hope to see

$$\frac{20,000}{\pi} = 6,366.2 \text{ crosses.}$$

That's impossible – you can't have two tenths of a cross. If you were *unbelievably lucky* and crossed exactly 6,366 times you would estimate $\pi \approx 3.1417$ – only three decimal places right. If you were just 10 crosses out of ten thousand shy of the hoped for number you would estimate $\pi \approx 3.147$ so even the second decimal place would round wrong. If you had 100 too few – just one percent of the ten thousand – you'd guess $\pi \approx 3.2$ – not even one decimal place!