Summer Math Play

Ethan Bolker
Debbie Borkovitz

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Searching

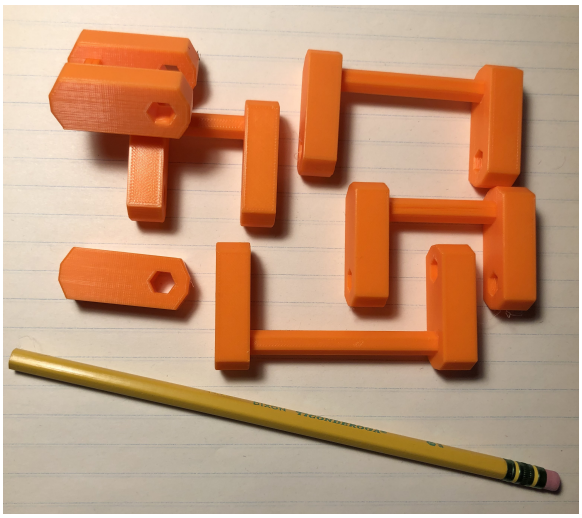
If this were a summer introduction to field paleontology and we were professional paleontologists we would find an unexplored Pacific island we thought a promising site for a student dig. We would look around just a bit, to see what might be discoverable — resisting the temptation to explore for ourselves.

There should be potentially many fossil treasures to find, in different environments. Some just lying around on the beach, some perhaps in impenetrable jungles inaccessible in a summer's work.

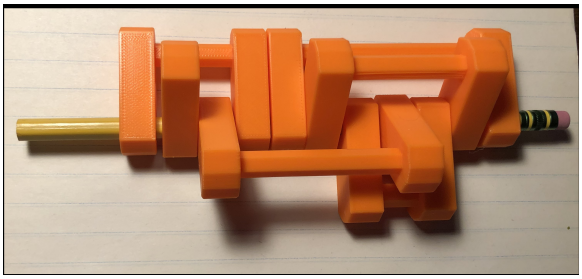
We think/hope we have found such a mathematical island. Your summer fun will be the exploration. Ours will be helping you — pointing you to tools for the job, suggesting directions and strategies when you're stuck. This isn't homework, where we know the answers to the questions. We don't even know the questions.[1]

Plug puzzle

This is a picture of six plugs. Five have two prongs, one has just one. In the two prong plugs the spaces between the prongs are multiples of the width of a prong. The values are $1, 2, 3, 4$ and 5.



You assemble the puzzle by threading the plugs onto a pencil with no gaps. This is one way to do that:



[1] Ann Hulsing suggested the paleontology analogy.

How hard is it to count the solutions for the version of this puzzle with n plugs? This puzzle was inspired by the question *Number of ways to arrange pairs of integers with distance constraint* at `http://math.stackexchange.com/questions/4124452/number-of-ways-to-arrange-pairs-of-integers-with-distance-constraint`.

There's no answer there, and we think the question is hard. But the island is promising since there are similar questions that are easy. For example,

- If you have n different single prong plugs there will be $n!$ solutions. If you have n identical single prong plugs there is just one solution.
- If you have three two prong plugs each with a single prong width separation there is no solution.

Tools

Paleontologists dig with picks and trowels and dirt screens along with a growing sense of what they are looking for. We'll use different tools.

- Curiosity
- People to bounce ideas off
- Writing down what we learn
- Discrete mathematics, a little number theory, perhaps some linear algebra
- Programming
- Mathematics Stackexchange
- The On-Line Encyclopedia of Integer Sequences
- LaTeX

Here is the LaTeXwe compiled to produce this pdf:

```
% Notes for the start of an informal summer research experience
% for a few BU students.
%
% Ethan Bolker and Debbie Borkovitz
%
\documentclass[10pt]{article}
\usepackage[textheight=10in, textwidth=7in]{geometry}

\usepackage{amsmath}
\usepackage{amsthm}
\usepackage{amsfonts} % to get \mathbb letters
\usepackage{xcolor}
\usepackage{array}
\usepackage{graphicx}
\usepackage{tabularx}% http://ctan.org/pkg/tabularx
\usepackage{booktabs}% http://ctan.org/pkg/booktabs
\usepackage{hyperref}
\usepackage{verbatim}

\newtheorem{theorem}{Theorem}
\newtheorem{definition}[theorem]{Definition}
\newtheorem{lemma}[theorem]{Lemma}
\newtheorem{conjecture}[theorem]{Conjecture}
\newtheorem{corollary}[theorem]{Corollary}
\newtheorem{remark}[theorem]{Remark}
\newtheorem{question}{Question}
\newtheorem{example}[theorem]{Example}
\numberwithin{equation}{section}

\title{Summer Math Play}
\author{ Ethan Bolker\\Debbie Borkovitz}
\date{May, 2021}

\begin{document}

\maketitle

\section*{Searching}

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professional paleontologists we would find an unexplored Pacific island
we thought a promising site for a student dig.
We would look around just a bit, to see what might be
discoverable --- resisting the temptation to explore for ourselves.

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different environments. Some just lying around on the beach, some
perhaps in impenetrable jungles inaccessible in a summer's work.

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pointing you to tools for the job,
suggesting directions and strategies when you're stuck. This isn't
homework, where we know the answers to the questions. We don't even
know the questions.%
\footnote{Ann Hulsing suggested the paleontology analogy.}

\section*{Plug puzzle}
```

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This is a picture of six plugs. Five have two prongs, one has just
one. In the two prong plugs the spaces between the prongs are
multiples of the width of a prong. The values are $1, 2, 3, 4$ and
$5$.

\begin{center}
\includegraphics[width=3in]{orange.jpg}
\end{center}

You assemble the puzzle by threading the plugs onto a pencil with no gaps.
This is one way to do that:

\begin{center}
\includegraphics[width=3in]{orangeassembled1.jpg}
\end{center}

How hard is it to count the solutions for the
version of this puzzle with $n$ plugs?
This puzzle was inspired by the question
\emph{Number of ways to arrange pairs of integers with distance
  constraint}
at
\url{http://math.stackexchange.com/questions/4124452/number-of-ways-to-arrange-pairs-of-integers-with-dista

There's no answer there, and we think the  question is hard. But the
island is promising since there are similar questions that are easy.
For example,

\begin{itemize}
\item If you have $n$
different single prong plugs there will be $n!$ solutions. If you have
$n$ identical single prong plugs there is just one solution.

\item
If you have three two prong plugs each with a single prong width
separation there is no solution.
\end{itemize}

%\section{Notation suggestions}
%
%We will have to talk about plugs to each other and in writing.
%
\section*{Tools}

Paleontologists dig with picks and trowels and dirt screens along with
a growing sense of what they are looking for. We'll use different
tools.

\begin{itemize}
\item Curiosity
\item People to bounce ideas off
\item Writing down what we learn
\item Discrete mathematics, a little number theory, perhaps some linear algebra
\item Programming
\item  \href{math.stackexchange.com/}{Mathematics Stackexchange}
\item \href{oeis.org/}{The On-Line Encyclopedia of Integer Sequences}
\item \LaTeX
\end{itemize}

\newpage
```

```
Here is the \LaTeX we compiled to produce this pdf:

\verbatiminput{\jobname}
\end{document}
```